**CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE**

# Entrance Exam Wiskunde B

Date: 17 July 2021
Time: 140 minutes (2 hours and 20 minutes)
Questions: 4

**Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.**

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

*Because the time for this exam has been reduced to 140 minutes, the number of questions has been reduced. Therefore, the total number of points that can be scored is reduced to 72.*

| Points that can be scored for each item: | | | | |
|---|---|---|---|---|
| Question | 1 | 2 | 3 | 4 |
| a<br>b<br>c<br>d<br>e | 5<br>5<br>4<br>5<br>5 | 5<br>4<br>7 | 3<br>2<br>6<br>3 | 4<br>6<br>8 |
| Total | 24 | 16 | 14 | 18 |
| Grade $= \frac{\text{total points scored}}{8} + 1$<br>You will pass the exam if your grade is at least 5.5 . | | | | |

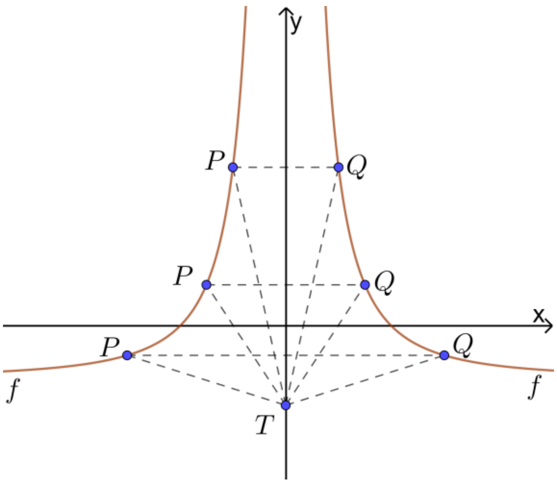
## Question 1 – A function of degree –2

*Take a new answer sheet for every question!*

The function $f$ is given by

$$f(x) = \frac{4}{x^2} - 1$$

At altitude $h$, with $h > -1$, there are horizontal line segments $PQ$ with $P$ on the left half of the graph of $f$ and $Q$ on the right half. Together with the point $T\left(0, -1\frac{1}{2}\right)$ these points form a triangle $PQT$ for each value of $h$.
In the figure below, three such triangles are shown.




The length $L$ of a line segment $PQ$ is given by

$$L = \frac{4}{\sqrt{h+1}}$$

and the area $A$ of a triangle $PQT$ is given by

$$A = \frac{2h+3}{\sqrt{h+1}}$$

5pt a Show that this is true.

5pt b Compute exactly the value of $h$ for which the area of the corresponding triangle $PQT$ is minimal.

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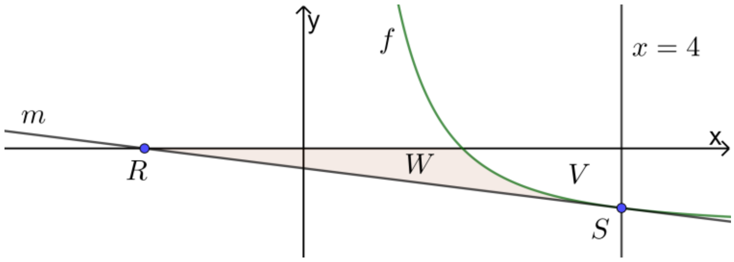
## Continuation of question 1

In the figure below, the right half of the graph of $f(x) = \frac{4}{x^2} - 1$ is shown.
$S$ is the intersection of this graph with the line $x = 4$ and *m* is the straight line through point $S$ and point $R(-2,0)$.
*V* is the region enclosed by the graph of $f$, the *x*-axis and the line $x = 4$.
*W* is the region enclosed by the graph of $f$, the *x*-axis and line *m*.




Line *m* is the tangent line to the graph of $f$ in point $S$.

4pt c Show that this is true.

*V* is rotated around the *x*-axis. The volume of the solid of revolution that is formed in this way is exactly equal to $\frac{7}{12}\pi$.

5pt d Show that this is true.

5pt e Compute exactly the volume of the solid of revolution that is formed by rotating *W* around the *x*-axis.

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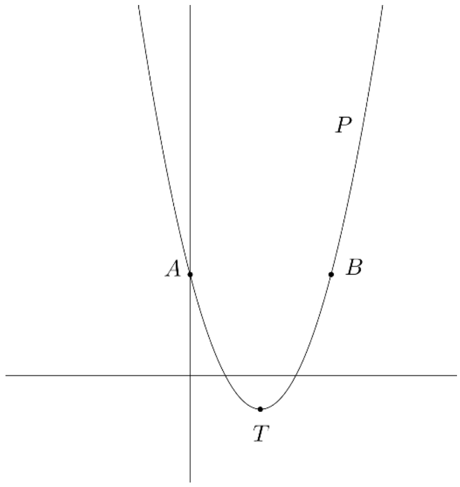
## Question 2 – Parabola, circles, line and triangle

*Take a new answer sheet for every question!*

Given is the parabola *P* with equation
$y = x^2 - 4x + 3$.
Point *T* is the vertex of the parabola.
*A* and *B* are the points on *P* for which $y = 3$.
See the figure on the right.




5pt a Use an exact computation to determine an equation for the circle through the points *A*, *B* and *T*.

For a second circle it is given that the centre is on the *x*-axis and that this circle touches the parabola *P* in point *A*.

4pt b Compute exactly the *x*-coordinate of the centre of this circle.

The line *m* is given by the vector representation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
On line *m* there are points *C* for which triangle *ABC* is right angled.

7pt c Compute exactly the coordinates of these points.

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## Question 3 – Functions with logarithms

*Take a new answer sheet for every question!*

For each real value of $k$, the function $f_k$ is given by

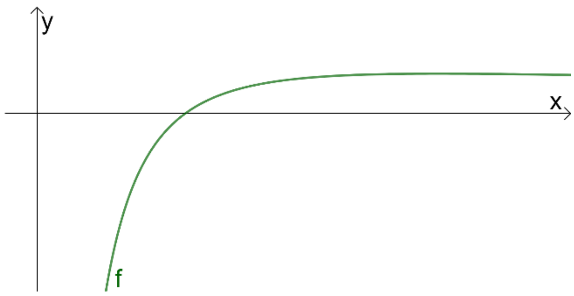
$$f_k(x) = \frac{2\ln(x) - 1}{x - k}.$$

There is one value of $k$ for which the graph of $f_k$ has a perforation (that is a removable discontinuity).

3pt a Compute exactly this value of $k$.

In the remainder of this question we take $k = 0$.

In the figure below the graph is shown of the function $f_0(x) = \frac{2\ln(x)-1}{x}$.




The function $F(x) = \ln^2(x) - \ln(x)$ is an antiderivative of $f_0$.

2pt b Show that this is true.

For each $a > 0$, $V_a$ is the region enclosed by the graph of $f_0$, the vertical line $x = a$ and the *x*-axis.

6pt c Compute exactly the values of $a$ for which the area of region $V_a$ equals 4.

Furthermore, the functions $g$ and $h$ are given by

$$g(x) = 3 - \ln\left(\frac{2}{x}\right) \quad \text{and} \quad h(x) = x \cdot f_0(x) - g(x)$$

The function rule of $h$ can be rewritten into the form $h(x) = \ln(bx)$.

3pt d Compute exactly the value of $b$.

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## Question 4 – Limaçon of Pascal

*Take a new answer sheet for every question!*

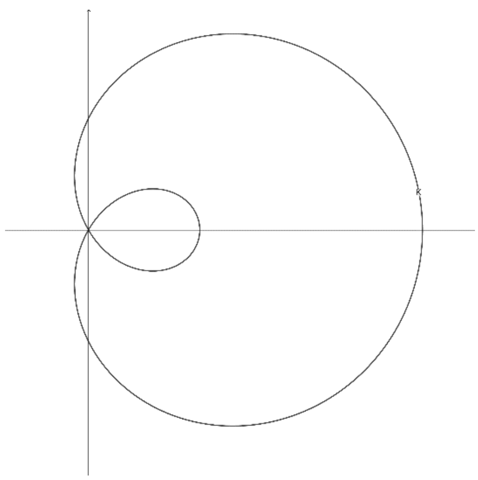
A curve given by a parametric representation of the form

$$\begin{cases} x = (a + b\cos(t)) \cdot \cos(t) \\ y = (a + b\cos(t)) \cdot \sin(t) \end{cases}$$

with $0 \leq t \leq 2\pi$, $a \neq 0$ and $b \neq 0$ is called a *Limaçon of Pascal*.
(*Named after Étienne Pascal, Blaise's father, who was the first to extensively explore these curves.*)
The shape of such a curve depends on the values of $a$ and $b$.

We take $a = 1$ and $b = 2$, so the parametric representation becomes:

$$\begin{cases} x = (1 + 2\cos(t)) \cdot \cos(t) \\ y = (1 + 2\cos(t)) \cdot \sin(t) \end{cases}$$



In the figure on the right the corresponding curve is shown. This curve intersects itself in the origin $O(0,0)$.

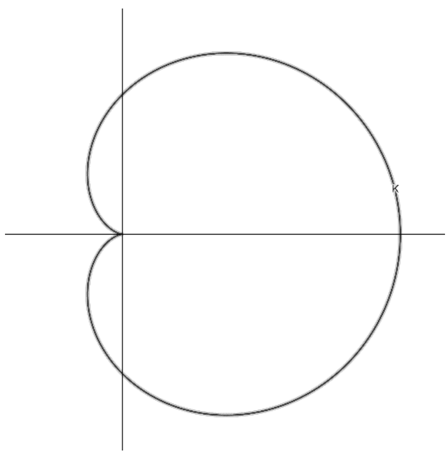
4pt a Compute exactly the values of $t$ belonging to this point.

The curve also intersects the *y*-axis for $t = \frac{1}{2}\pi$.

6pt b Compute algebraically the angle at which the curve intersects the *y*-axis for $t = \frac{1}{2}\pi$. Give your answer in degrees.

Taking $a = b = 1$, the parametric representation becomes

$$\begin{cases} x = (1 + \cos(t)) \cdot \cos(t) \\ y = (1 + \cos(t)) \cdot \sin(t) \end{cases}$$



The corresponding curve is again shown in the figure on the right.

8pt c Compute exactly the coordinates of the highest and the lowest point on this curve.

*End of the exam.*

*When you have finished the exam, check whether your* ***name*** *and the* ***question number*** *are on every answer sheet.*

*Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.*

*What should* ***not*** *be in the folder:*
*- empty sheets, please leave them on your table;*
*- sheets with only your name on it, please take them with you;*
*- scrap paper;*
*- these questions.*

*This is the only way we can ensure a smooth correction of your exam work.*

*Remain seated until one of the invigilators collects your folder (or calls you).*

## Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t+u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t-u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t+u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t-u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2\sin(t)\cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$$

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