CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date:	21 July 2020
Time:	3 hours
Questions:	6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:

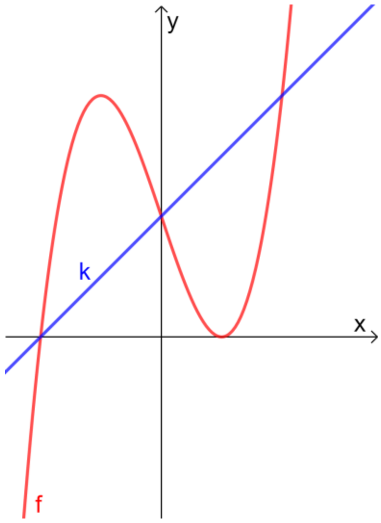
Question	1	2	3	4	5	6
a	4	4	3	8	4	7
b	5	6	6	4	7	8
c	8	5			4	7
Total	17	15	9	12	15	22

$$\text{Grade} = \frac{\text{total points scored}}{10} + 1$$

You will pass the exam if your grade is at least 5.5 .

Question 1

Given is the function $f(x) = (x-1)^2(x+2)$.
In the figure below, the graph of f and the line *k* with equation $y = x + 2$ are shown.



The graph of f and line *k* have three points in common.

4pt a Compute exactly the coordinates of these intersection points.

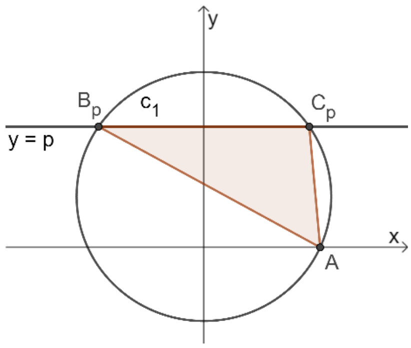
A horizontal line with equation $y = p$ also intersects the graph of f three times.

5pt b Compute exactly the values of p for which this is the case.

V is the bounded region enclosed by the graph of f and the *x*-axis. Line *k* divides this region into two parts.

8pt c Use an exact computation to show that the ratio of the areas of these two parts is 16 : 11.

Question 2



In the figure on the right, the circle c_1 with equation $x^2 + y^2 - 4y = 20$ is shown.
Point A is the intersection of circle c_1 and the positive *x*-axis.
For $p > 0$, points B_p and C_p are the intersections of circle c_1 and the horizontal line $y = p$.

The area of triangle AB_pC_p is equal to $p \cdot \sqrt{-p^2 + 4p + 20}$.

4pt a Show that this is true.

6pt b Compute exactly the maximal area of triangle AB_pC_p.

There are two values of p for which triangle AB_pC_p is a right angled triangle.

5pt c Compute exactly the length of the hypotenuse of triangle AB_pC_p for these two values of p.

Question 3

Given are the line ℓ with vector representation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and the circle c_2 with equation $(x-3)^2 + y^2 = 20$.
The lines m and n are the tangent lines to circle c_2 that are perpendicular to line ℓ.

3pt a Determine an equation of the line through the origin $(0,0)$ which is perpendicular to line ℓ.

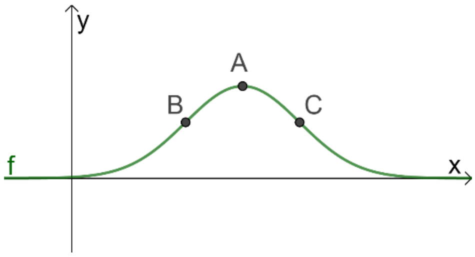
6pt b Compute exactly the coordinates of the points where the lines m and n are touching circle c_2.

Question 4

In the figure below, the graph is shown of the function

$$f(x) = \mathrm{e}^{-\frac{x^2}{2} + 3x - 4}$$

The graph of f has a maximum in point *A*.
B and *C* are the points of inflection of the graph of f.



8pt a Compute exactly the area of triangle *ABC*.

Furthermore given is the family of functions $g_a(x) = \mathrm{e}^{x-a}$.

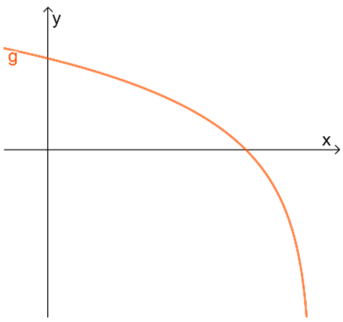
4pt b Compute exactly the value(s) of a for which the graph of f and the graph of g_a have exactly one point in common.

Question 5

Given is the family of functions $f_a(x) = \ln(x^2 + ax + 36)$.

4pt a Compute exactly the values of a for which the graph of f_a has two vertical asymptotes.

In the figure on the right, the graph is shown of the function $g(x) = \ln(4 - x)$.
V is the region enclosed by the graph of g, the *x*-axis and the *y*-axis.



7pt b Compute exactly the volume of the solid of revolution that is formed by rotating *V* round the *y*-axis.

Furthermore given is the family of functions $h_a(x) = \dfrac{f_a(x)}{g(x)}$.

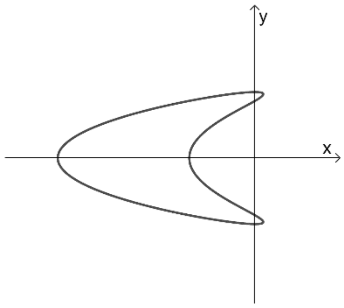
There is one value of a for which the graph of h_a has a perforation (that is a removable discontinuity).

4pt c Compute exactly this value of a.

Question 6

The movement of a point *P* is given by the parametric equations

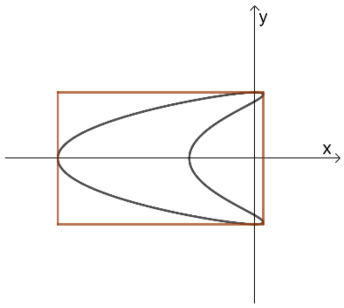
$$\begin{cases} x(t) = \sin(t) - 2\sin^2(t) \\ y(t) = \cos(t) \end{cases}$$



In the figure on the right, the path of point *P* is shown.

7pt a Compute exactly the *y*-coordinates of the points on the path of *P* of which the *x*-coordinate is equal to -1 .

The path of point *P* is enclosed by a rectangle, as shown in the figure on the right. As you can see, the path of *P* touches the sides of the rectangle in five points.



8pt b Compute exactly the area of this rectangle.

On $t = \frac{1}{4}\pi$, the path of *P* intersects line ℓ with equation $y = x + 1$.

7pt c Compute algebraically the angle between the path of *P* and line ℓ in this point. Give your answer in degrees, rounded to the nearest integer.

End of the exam.

Is your name on all answer sheets?

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2\sin(t)\cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$

