CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 22 July 2019
Time: 13.30 – 16.30 hours
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:

Question	1	2	3	4	5	6
a	6	6	3		6	6
b	4	6	2		6	4
c	2		4		7	6
d	6		5			5
Total	18	12	14	6	19	21

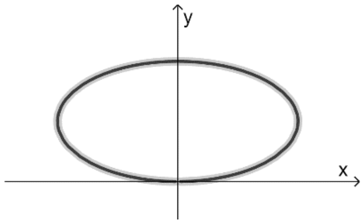
$$\text{Grade} = \frac{\text{total points scored}}{10} + 1$$

You will pass the exam if your grade is at least 5.5 .

Question 1

The movement of a point *P* is given by the parametric equations

$$\begin{cases} x(t) = \dfrac{4t}{1+t^2} \\ y(t) = \dfrac{2}{1+t^2} \end{cases}$$



In the figure on the right, the path of point *P* is shown.

6pt a Compute exactly the coordinates of the points where the tangent line to the path of *P* is vertical.

C is the curve with equation $x^2 + 4y^2 = 8y$.

4pt b Show that point *P* is on curve *C* for all values of t.

There is one point on curve *C* that is not on the path of point *P*.

2pt c Determine the coordinates of this point (explain your answer!)

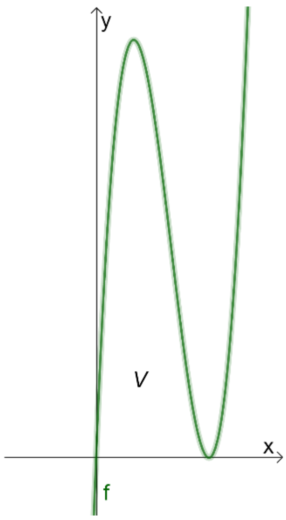
Furthermore, line ℓ is given with equation $y = x$.

6pt d Compute algebraically the angle between the path of *P* and line ℓ at their intersection.

Question 2

In the figure on the right, the graph is shown of the function

$$f(x) = 4x^3 - 20x^2 + 25x$$



V is the bounded region enclosed by the graph of f and the *x*-axis.

6pt a Compute exactly the area of region *V*.

6pt b Compute exactly the values of a and b for which the graph of the function $g(x) = \sqrt{ax+b}$ and the graph of f are touching in a point on the vertical line $x = 1$.

Question 3

Given are the points $A(4,3)$ and $B(1,7)$.
m is the straight line through the origin $O(0,0)$ and point A.
n is the line with vector representation $\binom{x}{y} = \binom{7}{-1} + \lambda \binom{-3}{4}$.

3pt a Show that lines *m* and *n* are perpendicular.

2pt b Show that point A and point B are both on line *n*.

c_1 is the circle that passes through points A and B and through the origin $O(0,0)$.

4pt c Compute the coordinates of the centre of circle c_1 .

c_2 is the circle with the following properties:

- The centre of c_2 is on the positive *x*-axis.
- The radius of c_2 is 4.
- The lines $y = 2x$ and $y = -2x$ are both tangent lines of circle c_2 .

5pt d Compute the *x*-coordinate of the centre of circle c_2 .

Question 4

The function f is given by $f(x) = \mathrm{e}^{1/x}$, that is $f(x) = \mathrm{e}^{g(x)}$ with $g(x) = \frac{1}{x}$.
The graph of f has one point of inflexion.

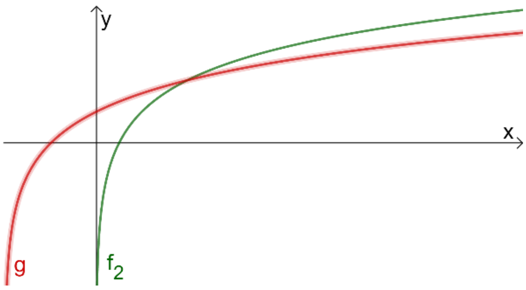
6pt Compute exactly the coordinates of this point of inflexion.

Question 5

For each $a > 0$, the function f_a is given by $f_a(x) = \ln(ax)$.
P_a is the intersection of the graph of f_a with the *x*-axis.
ℓ_a is the straight line that intersects the graph of f_a perpendicularly in point P_a .

6pt a Compute exactly the value(s) of a for which the line ℓ_a intersects the *y*-axis in point $(0,4)$.

In the figure on the right, the graphs are shown of the functions $f_2(x) = \ln(2x)$ and $g(x) = \ln(2 + x)$.
For each $p > 0$, F_p is the point $(p, f_2(p))$ and G_p is the point $(p, g(p))$.



6pt b Compute exactly the value(s) of p for which the distance between points F_p and G_p is equal to 1.

V is the region enclosed by the graph of g, the *x*-axis and the *y*-axis.
S is the solid of revolution that is formed by revolving *V* round the *y*-axis.

7pt c Compute exactly the volume of *S*.

Question 6 is on the next page!

Question 6

For $0 \leq x \leq 2\pi$, the function f is given by

$$f(x) = \frac{\cos(x)}{\cos(x) + \sin(2x)}$$

6pt **a** Compute exactly the coordinates of the perforations of the graph of f (these are the points where the graph of f has a removable discontinuity) on the interval $[0,\ 2\pi]$.

4pt **b** Compute exactly the equation(s) of the vertical asymptote(s) of the graph of f on the interval $[0,\ 2\pi]$.

The function g is given by $g(x) = \cos(2x) + \sin\left(3x + \frac{1}{3}\pi\right)$.

6pt **c** Compute exactly the *x*-coordinates of the intersections of the graph of g and the *x*-axis on the interval $[0,\ 2\pi]$.

Point *A* is the intersection of the graph of g and the *y*-axis.
ℓ is the tangent line to the graph of g at point *A*.
Point *B* is the intersection of line ℓ and the *x*-axis.

5pt **d** Compute exactly the *x*-coordinate of point *B*.

End of the exam.

Is your name on all answer sheets?

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2\sin(t)\cos(t)$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$$

Answers

1a $(2{,}1)$ and $(-2{,}1)$

1b $\left(x(t)\right)^2 + 4\left(y(t)\right)^2 = \dfrac{16t^2}{(1+t^2)^2} + \dfrac{4 \cdot 4}{(1+t^2)^2} = \dfrac{16(t^2+1)}{(1+t^2)^2} = \dfrac{16}{1+t^2} = 8y$

1c The origin $(0{,}0)$ lis on *C*, but $y(t) \neq 0$, so the origin is not on the path of *P*

1d $78.69°$

2a $\frac{625}{48}$

2b $a = -54;\ b = 135$

3a Two lines are perpendicular if the inner product of their direction vectors is 0.
The direction vector of *m* is $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and the direction vector of *n* is $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$\left(\begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix}\right) = 4 \cdot -3 + 3 \cdot 4 = 0$

3b $\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \end{pmatrix} \Leftrightarrow \begin{cases} 4 = 7 - 3\lambda \\ 3 = -1 + 4\lambda \end{cases} \Rightarrow \lambda = 1$ is OK

$\begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \end{pmatrix} \Leftrightarrow \begin{cases} 1 = 7 - 3\lambda \\ 7 = -1 + 4\lambda \end{cases} \Rightarrow \lambda = 2$ is OK

3c $\left(\frac{1}{2}, 3\frac{1}{2}\right)$

3d $\sqrt{20}$

4 $x = -\frac{1}{2};\ y = \mathrm{e}^{-2}$

5a $a = \frac{1}{2}$

5b $p = \frac{2}{2\mathrm{e}-1}$

5c $\pi \cdot \left(-2\frac{1}{2} + 4\ln(2)\right)$

6a $\left(\frac{1}{2}\pi, \frac{1}{3}\right)$ and $\left(1\frac{1}{2}\pi, -1\right)$

6b $x = 1\frac{1}{6}\pi$ and $x = 1\frac{5}{6}\pi$

6c $\frac{7}{30}\pi,\ \frac{19}{30}\pi,\ \frac{31}{30}\pi,\ \frac{43}{30}\pi,\ \frac{11}{6}\pi$ and $\frac{7}{6}\pi$

6d $x_B = -\frac{2}{3} - \frac{1}{3}\sqrt{3}$

