CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 19 December 2018
Time: 13.30 – 16.30 hours
Questions: 5

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in reduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put in your bag.

Points that can be scored for each question:					
Question	1	2	3	4	5
a	6	3	6	5	5
b	4	5	3	7	6
c	6	5	7	5	7
d			6	4	
Total	16	13	22	21	18

$\text{Grade} = \dfrac{\text{total points scored}}{10} + 1$

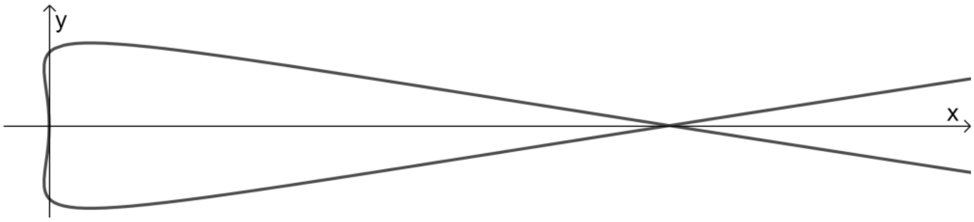
You will pass the exam if your grade is at least 5.5 .

Question 1

The movement of a point *P* is given by the parametric equations

$$\begin{cases} x(t) = 2t^4 - t^2 \\ y(t) = t^3 - 3t \end{cases}$$

In the figure below, the path of point *P* is shown.



6pt a Compute exactly the coordinates of the points where the path of *P* has a vertical tangent line.

4pt b Compute exactly the velocity (that is the length of the velocity vector) of point *P* on $t = 2$.

6pt c Compute the angle at which the path of *P* intersects itself on the positive *x*-axis. Give your answer rounded to whole degrees.

Question 2

In this question, we consider the circles c_1 and c_2 and the points $A(0,5)$, $B(2,5)$ and $C(14,5)$. Circle c_1 has equation $x^2 + y^2 + 2y - 15 = 0$; line segment *BC* is a diameter of circle c_2 .

3pt a Determine an equation for circle c_2 .

5pt b Prove that c_1 and c_2 are touching circles.

There are two lines through point *A* that are tangent lines to circle c_1 .

5pt c Find equations for these two tangent lines.

Question 3

Given is the function

$$f(x) = \frac{3x^2 - 6x}{(2x+1)(x^2-4)}$$

The graph of f has two vertical asymptotes, a horizontal asymptote and a perforation (that is a removable discontinuity).

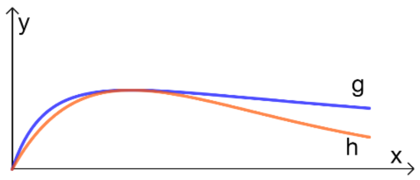
6pt a Find the equations of the three asymptotes of the graph of f.

Furthermore given is the function

$$g(x) = \frac{2}{x+2} - \frac{1}{2x+1}$$

Except for the perforation of the graph of f, which is the point $\left(2, \frac{3}{10}\right)$, the graphs of f and g are equal.

3pt b Show algebraically that this is indeed the case.



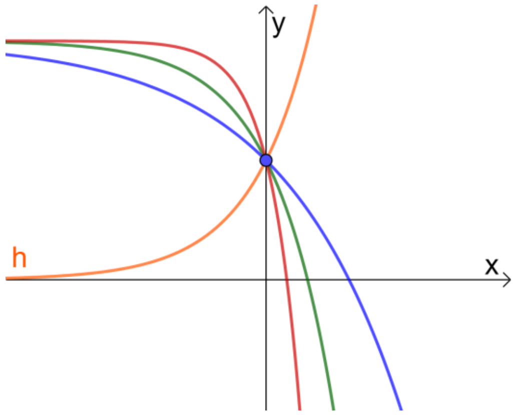
In the figure on the right, the graphs of g and of the function $h(x) = \frac{1}{3}x\mathrm{e}^{1-x}$ are shown for $0 \leq x \leq 3$. In the figure, these functions seem to have the same maximum on this interval.

7pt c Investigate algebraically whether this is true.

6pt d Compute exactly the area of the bounded region that is enclosed by the graph of g, the x-axis and the line $x = 2$. Simplify the answer as far as possible.

Question 4

For each *p*, the function g_p is given by $g_p(x) = 4 - 2\mathrm{e}^{px}$. Furthermore, we consider the function $h(x) = 2\mathrm{e}^{x}$. In the figure below, the graph of *h* is shown along with the graphs of g_p for three different values of *p*.



The graphs of the functions g_p and the graph of the function *h* all pass point $(0,2)$.

5pt a Compute the value of *p* for which the graph of g_p intersects the graph of *h* perpendicularly in point $(0,2)$.

The intersection of the graph of g_p and the vertical line $x = \ln(2)$ is called A_p .

7pt b Compute the value(s) of *p* for which the distance between A_1 and A_p equals 8.

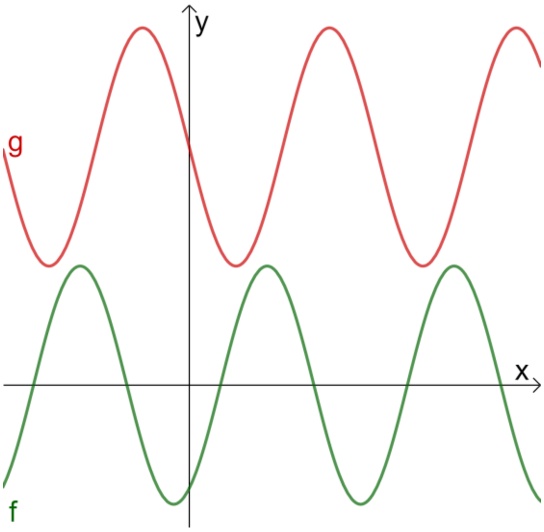
For $q > 0$, R_q is the region enclosed by the graph of g_1, the graph of g_2 and the vertical line $x = q$.

5pt c Show that the area of R_q equals $\mathrm{e}^{2q} - 2\mathrm{e}^{q} + 1$.

4pt d Compute the value of *q* for which the area of R_q equals 4.

Question 5

In the figure below, the graphs are shown of the functions $f(x) = 2\sin\left(2x - \frac{1}{3}\pi\right)$ and $g(x) = 4 - 2\sin(2x)$.



5pt a Compute the x-coordinates of the intersections of the graph of f and the line $y = -1$ on the interval $0 \leq x \leq 2\pi$.

The tangent line to the graph of f in point $A\left(\frac{1}{6}\pi, 0\right)$ intersects the *y*-axis in point *B*.

6pt b Compute the area of triangle *OAB*, where *O* is the origin $(0,0)$.

For each value of q, the vertical line $x = q$ intersects the graph of f in point F_q and intersects the graph of g in point G_q. $L(q)$ is the distance between points F_q and G_q .

7pt c Compute exactly the minimal value of $L(q)$.

End of the exam.

Is your name on all answer sheets?

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2\sin(t)\cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$

