CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Example Exam Wiskunde B

Date: Autumn 2018
Time: 3 Hours
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in reduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put in your bag.

Points that can be scored for each question:

Question	1	2	3	4	5	6
a	4	4	5	4	5	5
b	6	7	5	5	6	5
c		5	5	5	7	
d		7				
Total	10	23	15	14	18	10

$$\text{Grade} = \frac{\text{total points scored}}{10} + 1$$

You will pass the exam if your grade is at least 5.5 .

Question 1

Source: Centraal Examen vwo 2018 tijdvak 2

The function f is given by $f(x) = \frac{-2+2\sqrt{x+1}}{x}$.

Also given is the function h by $h(x) = \frac{2}{1+\sqrt{x+1}}$.

For $x \neq 0$ we have $f(x) = h(x)$

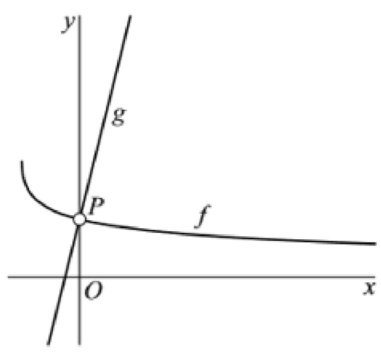
4pt a Prove that for $x \neq 0$ we indeed have $f(x) = h(x)$

Furthermore, the function g. is given by $g(x) = \frac{4x^2+x}{x}$.

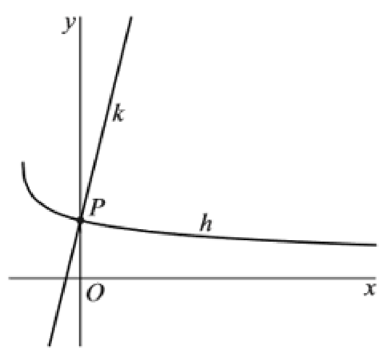
There is a straight line *k* that for $x \neq 0$ coincides with the graph of g .

In *figuur 1* the graphs of f and g are shown.
Point $(0,1)$ is the perforation of both graphs.
In *figuur 2* the graph of h, line *k* and their intersection *P* are shown.

figuur 1



figuur 2



We have:

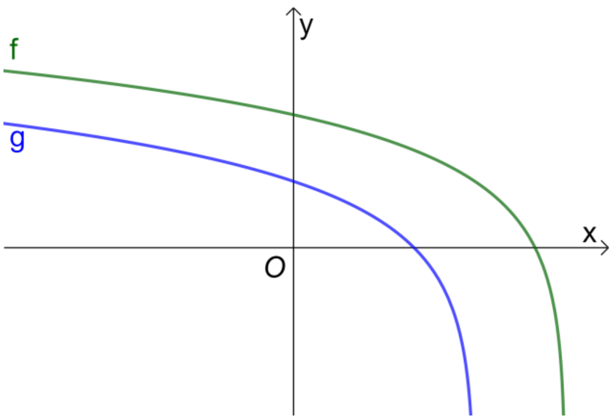
> The graphs of f and g are perpendicular in their perforation *P* if the graph of h and line *k* are perpendicular in their intersection *P*.

6pt b Prove that the graphs of f and g are perpendicular in their perforation *P*.

Question 2

Source: CCVX entrance exam wiskunde B July 2018

In the figure below, the graphs are shown of the functions $f(x) = \ln(9 - 2x)$ and $g(x) = \ln(3 - x)$.



The vertical line $x = p$ intersects the graph of f in point *P* and the graph of g in point *Q*. The distance between point *P* and point *Q* equals $\ln(4)$.

4pt a Use an exact computation to find the value of p .

The graph of f intersects the *y*-axis in point *R*. The graph of g intersects the *y*-axis in point *S*. The tangent to the graph of f in point *R* and the tangent to the graph of g in point *S* intersect in point *T*.

7pt b Compute exactly the *x*-coordinate of point *T* and simplify the answer.

Furthermore, the function h is given by $h(x) = 2 \cdot \ln(x + 3)$.

5pt c Compute exactly the coordinates of the intersection(s) of the graphs of f and h .

V is the bounded region enclosed by the graph of g, the *x*-axis and the *y*-axis.

7pt d Compute exactly the volume of the solid of revolution that is formed by revolving *V* around the *y*-axis.

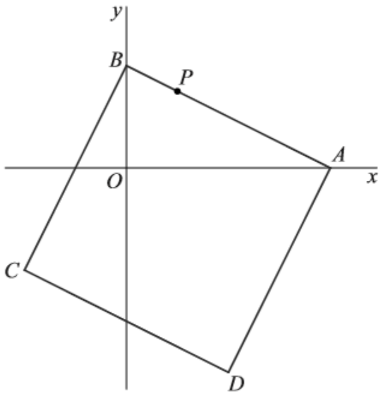
Question 3

Source: Centraal Examen vwo 2018 tijdvak 2

Given is the square $ABCD$ with vertices $A(8,0)$, $B(0,4)$, $C(-4,-4)$ en $D(4,-8)$.

Point $P(2,3)$ is on side AB.
See *figuur 1*.

figuur 1



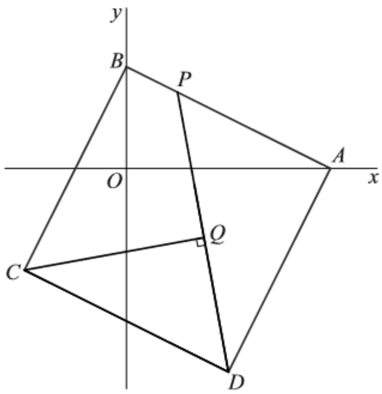
The points B, C and P are on one circle.

6pt a Find an equation for this circle.

A point Q is moving along line segment DP (from D to P).

There is a position of point Q for which line segment CQ is perpendicular to line segment DP.
See *figuur 2*.

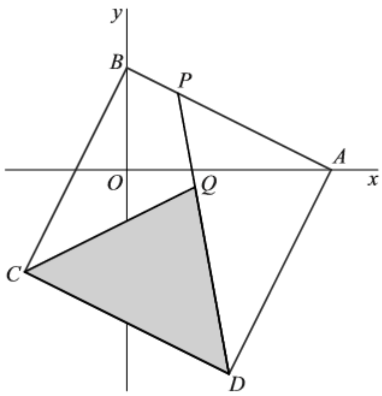
figuur 2



6pt b Compute exactly the coordinates of point Q when it is in this position.

In *figuur 3* triangle CDQ is shaded. There is a position of point Q for which the area of triangle CDQ is one third part of the area of square $ABCD$.

figuur 3

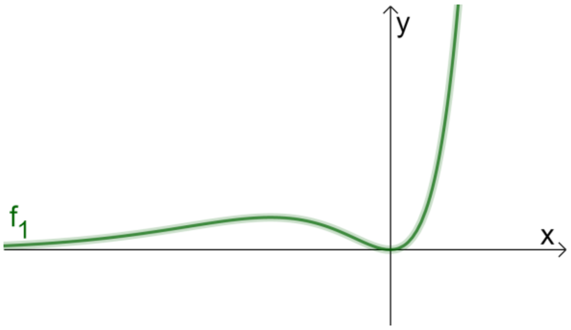


6pt c Compute exactly the coordinates of point Q when it is in this position.

Question 4

Source: CCVX entrance exam wiskunde B July 2018 (adjusted)

For each $a \neq 0$, the function f_a is given by $f_a(x) = x^2 \cdot \mathrm{e}^{ax}$.
In the figure below, the graph is shown of the function f_1 .



As you can see, the graph of f_1 has two points of inflection.

5pt a Compute exactly the *x*-coordinates of these two points of inflection.

Like f_1, all functions f_a have a minimum in (0,0). They also have a maximum. The point on the graph where f_a has a maximum is denoted by P_a . The points P_a are all on the same parabola.

6pt b Use an exact computation to find an equation for this parabola.

There are two tangent lines to the graph of f_2 with an equation of the form $y = px$, Therefore, these tangent lines pass through the origin (0,0).

7pt c Use an exact computation to find equations for these two tangent lines.

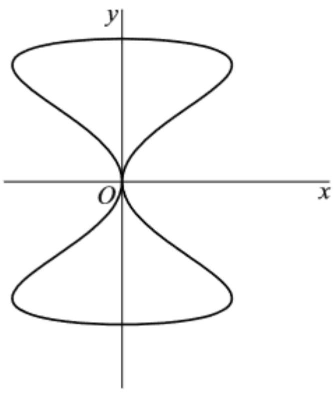
Question 5

Source: Centraal Examen vwo 2018 tijdvak 2

For $0 \leq t \leq 2\pi$, the position of a moving point is given by the parametric equations

$$\begin{cases} x(t) = \cos(t)\sin(2t) \\ y(t) = \cos(t) \end{cases}$$

figuur 1



The path of the moving point is shown in *figuur 1*.

At $t = \frac{1}{2}\pi$ and $t = 1\frac{1}{2}\pi$, the moving point is in *O*. This situation is disregarded in the entire question.

P_t is the position of the moving point at time *t*.

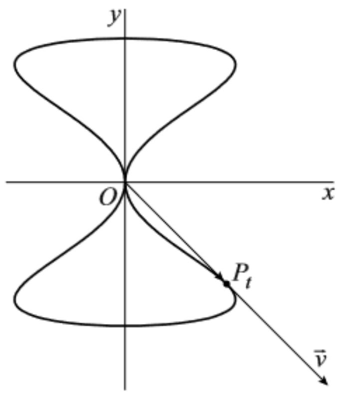
We have: The line through P_a and $P_{\pi - a}$ is vertical for every value of a that is possible in this situation.

4pt a Show that this line is indeed vertical.

There are several times at which the distance of P_t to the *x*-axis is twice the distance of P_t to the *y*-axis.

5pt b Compute exactly the fourth time at which this is the case.

figuur 2



For each value of *t*, the velocity vector $\vec{v}$ from point P_t and the vector $\overrightarrow{OP_t}$ can be drawn. In *figuur 2*, point P_t, vector $\overrightarrow{OP_t}$ and vector $\vec{v}$ are drawn for $t = \frac{3}{4}\pi$.

5pt c Show that for $t = \frac{3}{4}\pi$ we have: $\overrightarrow{OP_t} = \vec{v}$

Question 6

Source: New

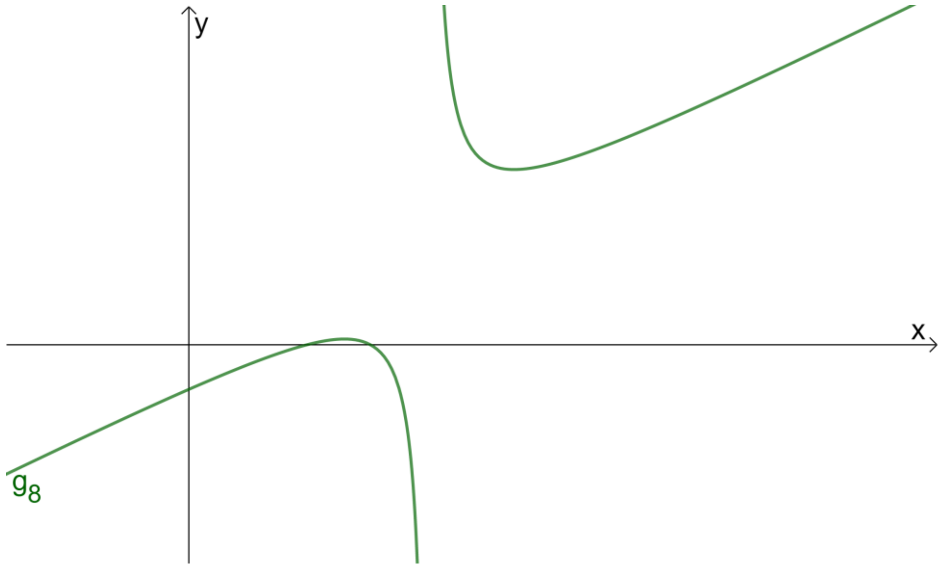
The family of functions g_p is given by

$$g_p(x) = \frac{x^2 - 5x + 6}{2x - p}$$

For $p \neq 4$, the graph of the function g_p intersects the parabola with equation $y = \frac{1}{4}x^2 - 1$ in the point (2,0).

5pt a Use an exact computation to determine the value(s) of p for which this parabola and the graph of g_p are touching in point (2,0).

In the figure below the graph is shown of the function g_8.
This graph has two asymptotes.



5pt b Use an exact computation to determine the equations of the two asymptotes of the graph of g_8 .

Extra item: What do the graphs of g_4 and g_6 look like?

End of the exam.

Is your name on all answer sheets?

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

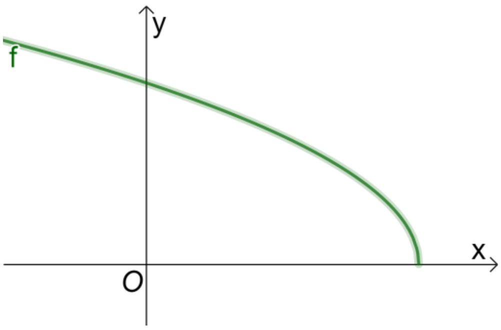
$\sin(2t) = 2\sin(t)\cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$

Extra question 1

Source: CCVX entrance exam wiskunde B July 2018

In the figure below the graph is shown of the function $f(x) = \sqrt{9-2x}$.



Line ℓ is the tangent line to the graph of f at the intersection with the *y*-axis.
T is the triangle enclosed by line ℓ , the *x*-axis and the *y*-axis.

6pt a Compute exactly the area of triangle *T*.

V is the bounded region enclosed by the graph of f, the *x*-axis and the *y*-axis.

6pt b Compute exactly the area of region *V*.

Furthermore, the function g is given by $g(x) = 2x + 11$.

5pt c Solve exactly the equation $f(x) = g(x)$.

Extra question 2

Source: CCVX entrance exam wiskunde B July 2018

The functions f and g are given by $f(x) = \sin^2(x) + \cos(2x)$ and
$g(x) = \sin^2(x) + \sin\left(2x - \frac{1}{6}\pi\right)$.

7pt a Solve the equation $f(x) = g(x)$ exactly and find all solutions in the interval $[-\pi, \pi]$.

7pt b Use the derivative function to find the minimal and the maximal value of $f(x)$ exactly.

The function h is given by $h(x) = f(x) + g(x) + 2\cos^2(x) - 2$,

4pt c Show that h can be written as $h(x) = \sin(2x + \frac{1}{6}\pi)$.

4pt d Compute exactly:

$$\int_0^{\frac{\pi}{2}} h(x)\ \mathrm{d}x$$