

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 17 December 2024

Time: 13.30 – 16.30

Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:				
Question	1	2	3	4
a	6	6	5	6
b	4	2	4	7
c	4	4	8	6
d	7	7	5	
Total	21	19	22	19
Grade = $\frac{\text{total points scored}}{9} + 1$				
You will pass the exam if your grade is at least 5.5 .				

Question 1 – Rational functions

Take a new answer sheet for every question!

The function f is given by

$$f(x) = \frac{x^2 + 3x - 10}{2x^2 - 3x - 2}$$

- 6pt a Compute exactly the coordinates of the intersection point of the tangent line to the graph of f in the point $(0,5)$ and the line with equation $x - 6y = 25$.
- 4pt b Compute exactly the value(s) of x for which $f(x) = 1$.

The function g is given by

$$g(x) = \frac{x^2 - 7x + 12}{x^3 - 3x^2}$$

The graph of g has one perforation (in some textbooks, this is called a removable discontinuity).

- 4pt c Compute exactly the coordinates of this perforation.

The family of functions h_a is given by

$$h_a(x) = \frac{10}{x^4 - 4x^2 + a}$$

- 7pt d Compute exactly the values of a for which the graph of h_a has four intersection points with the horizontal line $y = 5$.

Question 2 – A special family

Take a new answer sheet for every question!

The family of functions f_p is given by $f_p(x) = e^{x^2-2x} + p \cdot (x^2 - 2x)$.

Note that for all values of p we have:

$$f_p(0) = e^0 + p \cdot 0 = 1 + 0 = 1 \quad \text{and} \quad f_p(2) = e^0 + p \cdot 0 = 1 + 0 = 1.$$

Thus the graphs of all functions f_p pass through the points $A(0,1)$ and $B(2,1)$.

In question a we take $p = -e^3$. This yields $f_{-e^3}(x) = e^{x^2-2x} - e^3 \cdot (x^2 - 2x)$.

- 6pt a Compute exactly the coordinates of the points on the graph of f_{-e^3} where this function has an extreme value (minimum or maximum).

For all values of p , P_p is the point on the graph of f_p with $x_p = 1$.

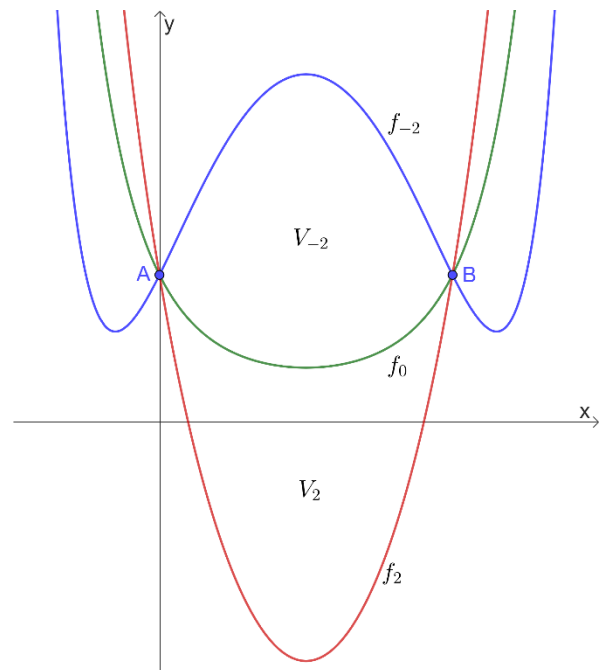
- 2pt b Use an exact computation to show that the tangent line to the graph of f_p is horizontal in point P_p for all values of p .

- 4pt c Compute exactly the value of p for which $f_p''(1) = 0$.

For all $p \neq 0$, the graphs of f_p and $f_0(x) = e^{x^2-2x}$ enclose a region V_p , that is located between the points $A(0,1)$ and $B(2,1)$.

As an example, the graphs of f_0 , f_{-2} and f_2 and the corresponding regions V_{-2} and V_2 are shown in the figure on the right.

- 7pt d Compute exactly the value(s) of p for which the area of region V_p is equal to 4.



Question 3 – Two logarithmic functions and two circles

Take a new answer sheet for every question!

The functions f and g are given by $f(x) = \ln(2x^2 - x)$
and $g(x) = 2 \cdot \ln(x - 2)$.

5pt a Solve the equation $f(x) = g(x)$ exactly.

The horizontal line $y = 1$ intersects the graph of f in the points P and Q .

4pt b Compute exactly the distance between the points P and Q .

Point A is the intersection of the graph of f and the positive x -axis.

Point B is the intersection of the graph of g and the x -axis.

Circle c_1 touches the graph of f in point A .

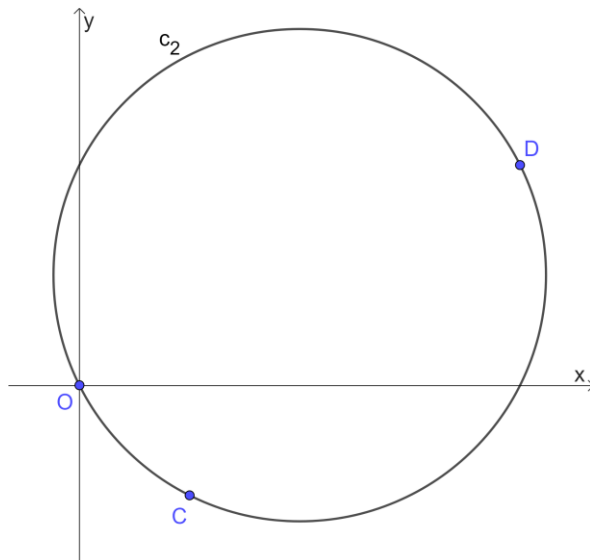
The centre M of circle c_1 is on the vertical line that passes through point B .

8pt c Compute exactly an equation for circle c_1 .

In the figure below, circle c_2 is shown. This circle passes through the origin $O(0,0)$
and through the point $C(2, -2)$. The radius of circle c_2 is $r = 2\sqrt{5}$.

The line segment OD is a diameter of circle c_2 .

5pt d Compute exactly the area of triangle OCD .



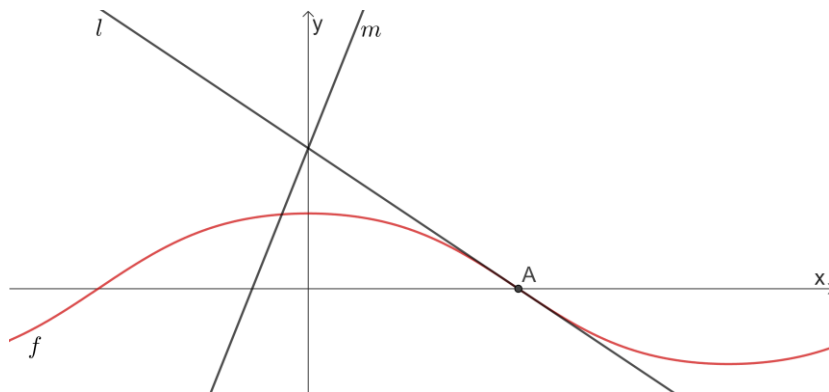
Question 4 – Two trigonometric functions and an equation

Take a new answer sheet for every question!

In the figure below, the graph of the function $f(x) = 2 \sin\left(\cos\left(\frac{1}{3}x\right)\right)$ is shown.

Line l is the tangent line to the graph of f in the point $A\left(\frac{3}{2}\pi, 0\right)$.

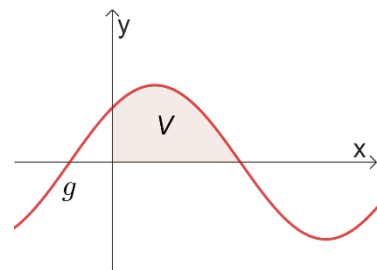
Line m is the line with vector representation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ \pi \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 5 \end{pmatrix}$.



- 6pt a Compute algebraically the angle between line l and line m .
Give your answer in degrees, rounded to one digit behind the decimal point.

In the figure on the right, a part of the graph of the function $g(x) = \sin(x) + \cos(x)$ is shown.

V is the region enclosed by the graph of g , the x -axis and the y -axis that is located on the right side of the y -axis.



- 7pt b Compute exactly the volume of the solid of revolution that is formed by rotating V around the x -axis.
- 6pt c Solve the equation $\sin(3x) = \cos(5x)$ exactly and give all solutions in the interval $0 \leq x \leq \pi$.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$