

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 23 July 2024
Time: 13.30 – 16.30
Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:				
Question	1	2	3	4
a	5	5	7	5
b	8	7	7	5
c	7	6	6	7
d		6		
Total	20	24	20	17
Grade = $\frac{\text{total points scored}}{9} + 1$				
You will pass the exam if your grade is at least 5.5 .				

Question 1 – Four logarithmic functions and a triangle

Take a new answer sheet for every question!

The function f is given by $f(x) = \ln(2x^2 + 7)$.

The function g is given by $g(x) = 2\ln(2x + 1)$.

- 5pt a Compute exactly the coordinates of the intersection(s) of the graphs of f and g .

The function h is given by $h(x) = \ln(x^2 + 4)$.

The function h has its minimum value in point A and the graph of h has two inflection points, B and C .

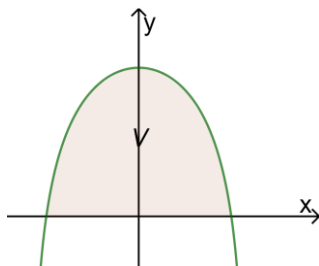
Triangle D is the triangle with corners A , B and C .

- 8pt b Use an exact computation to show that the area of triangle D is equal to $\ln(4)$.

In the figure below, the graph is shown of the function $k(x) = 2\ln(4 - x^2)$.

V is the region enclosed by the graph of k and the x -axis.

- 7pt c Compute exactly the volume of the solid of revolution that is formed by rotating V around the y -axis.



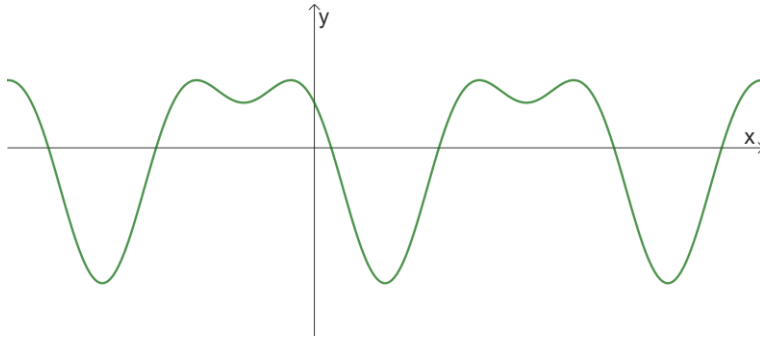
Question 2 – Four trigonometric functions

Take a new answer sheet for every question!

The function f is given by $f(x) = 2 \cos^2(x) - 5 \sin(x) - 5$.

5pt a Solve the equation $f(x) = 0$ exactly.

In the figure below, a part of the graph of the function $g(x) = \cos(2x) - 2 \sin(x)$ is shown.



7pt b Compute exactly the maximal and the minimal value of $g(x)$.

The function h is given by $h(x) = 6 \sin(3x)$.

6pt c Compute exactly all values of a in the interval $0 \leq a \leq \pi$ for which

$$\int_0^a h(x) \, dx = 1.$$

The function k is given by $k(x) = x^2 \cos\left(\frac{\pi}{x}\right)$.

Point P is the point on the graph of k for which $x_P = 2$.

Line ℓ is the tangent line to the graph of k in point P .

6pt d Compute algebraically the angle between line ℓ and the y -axis.
Give your answer in degrees, rounded to one digit behind the decimal point.

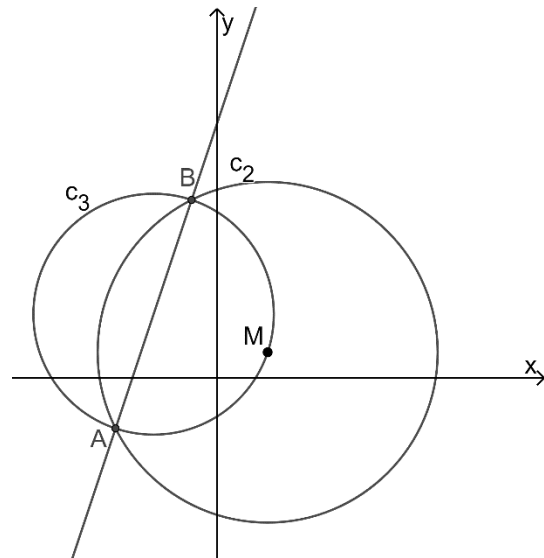
Question 3 – Circles and triangles

Take a new answer sheet for every question!

The function f is given by $f(x) = (x^2 - 2x) \cdot e^{x^2 - 2x}$.
 The graph of f intersects the x -axis in points P and Q .
 Circle c_1 touches the graph of f in both of these points.

7pt a Compute exactly an equation for circle c_1 .

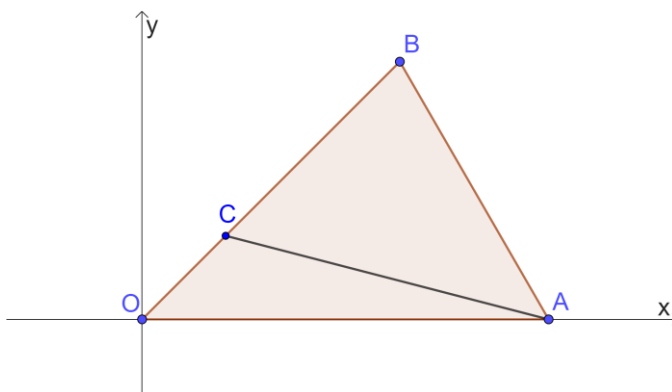
In the figure on the right, circle c_2 with equation $(x - 2)^2 + (y - 1)^2 = 45$ and the line l with equation $y = 3x + 10$ are shown. Points A and B are the intersections of circle c_2 and line l . Circle c_3 passes through points A and B and through the centre M of circle c_2 .



7pt b Use an exact computation to show that line segment AB is a diameter of circle c_3 .

In the figure below, a triangle is shown with corners $O(0,0)$, $A(3 + \sqrt{3}, 0)$ and $B(3,3)$. Point C is the point on side OB for which triangle ACB is similar to triangle OAB .

6pt c Compute exactly $\angle OAC$, the acute angle between AC and the x -axis.



Question 4 – Rational functions and power functions

Take a new answer sheet for every question!

The family of functions f_a is given by

$$f_a(x) = \frac{x^2 + x - 6}{2x - a}$$

There is one positive value of a for which the graph of f_a has a perforation (that is a removable discontinuity).

5pt a Compute exactly the coordinates of the perforation for this value of a .

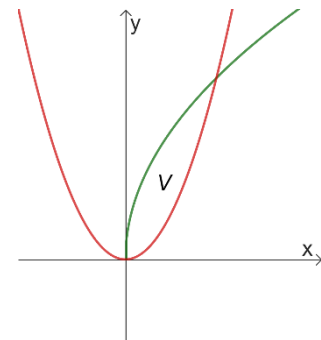
We now take $a = 6$.

5pt b Compute exactly equations for the asymptotes of the graph of f_6 .

In the figure on the right, the graphs are shown of the functions $g(x) = \sqrt{8x}$ and $h(x) = x^2$.

V is the region enclosed by the graphs of g and h .

7pt c Compute exactly the area of region V .



End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$