

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 22 April 2024

Time: 13.30 – 16.30

Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:				
Question	1	2	3	4
a	5	5	5	6
b	6	4	6	5
c	8	6	7	6
d		7	5	
Total	19	22	23	17
Grade = $\frac{\text{total points scored}}{9} + 1$				
You will pass the exam if your grade is at least 5.5 .				

Question 1 – Two logarithmic functions

Take a new answer sheet for every question!

The function f is given by $f(x) = 2 \cdot \ln\left(\frac{1}{2}x\right)$.

The function g is given by $g(x) = \ln(8 - x)$.

- 5pt a Compute exactly the coordinates of the intersection(s) of the graphs of f and g .

Point A is the intersection of the graph of f and the x -axis.

Line ℓ is the tangent line to the graph of f in point A .

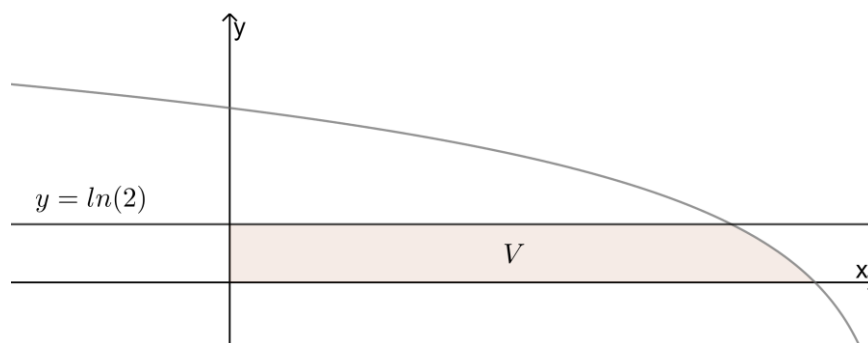
Point B is the intersection of line ℓ and the y -axis.

Triangle D is the triangle with corners A , B and $O(0,0)$.

- 6pt b Compute exactly the area of triangle D .

In the figure below, the graph of the function g is shown. V is the region enclosed by the graph of g , the x -axis, the y -axis and the horizontal line with equation $y = \ln(2)$.

- 8pt c Compute exactly the volume of the solid of revolution that is formed by rotating V around the y -axis.



Question 2 – Rational functions and functions with square roots

Take a new answer sheet for every question!

For each value of p , the function f_p is given by

$$f_p(x) = \frac{4x + 1}{2x^2 + 2x + p}$$

- 5pt a Compute exactly the value(s) of p for which the graph of f_p has two vertical asymptotes.

There is one value of p for which f_p has a maximum in a point on the y -axis.

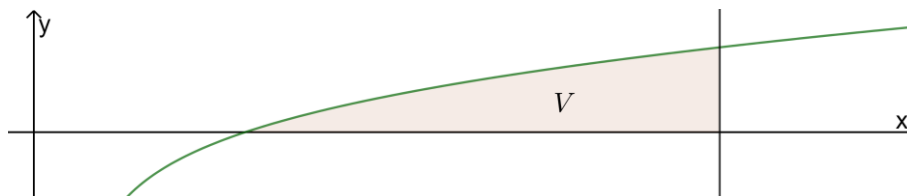
- 4pt b Compute exactly this value of p .

The function g is given by $g(x) = e^{-x} \cdot \sqrt{2x}$.

- 6pt c Compute exactly the maximal value of $g(x)$.

In the figure below, the graph is shown of the function $h(x) = \sqrt{3x - 3} - \sqrt{2x + 1}$. V is the region enclosed by the graph of h , the x -axis and the vertical line $x = 13$.

- 7pt d Compute exactly the area of region V .



Question 3 – The path of a moving point

Take a new answer sheet for every question!

The path of a moving point P is for any positive value of α given by

$$\begin{cases} x(t) = \cos(2t) \\ y(t) = 2 \sin(t) \cos(\alpha t) \end{cases} \quad \text{with } 0 \leq t \leq 2\pi.$$

There are four values of t in the interval $0 \leq t \leq 2\pi$ for which the path of P intersects with the vertical line $x = \frac{1}{2}$.

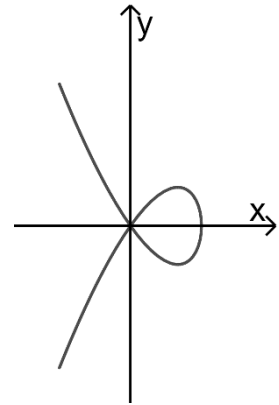
- 5pt a For the largest of these four values of t , compute exactly the values of α for which $y(t) = 1$.

For $\alpha = 2$ the path of point P is given by

$$\begin{cases} x(t) = \cos(2t) \\ y(t) = 2 \sin(t) \cos(2t) \end{cases} \quad \text{with } 0 \leq t \leq 2\pi.$$

See the figure on the right.

- 6pt b Compute exactly the coordinates of the intersections of the path of P and the line with equation $y = x$.



The path of point P in the figure on the right has two tangent lines in the origin $O(0,0)$.

- 7pt c Compute exactly an equation of one of these two tangent lines.

We now take $\alpha = 1$.

The path of P is then given by

$$\begin{cases} x(t) = \cos(2t) \\ y(t) = 2 \sin(t) \cos(t) \end{cases} \quad \text{with } 0 \leq t \leq 2\pi.$$

- 5pt d Compute exactly for which values of t in the interval $0 \leq t \leq 2\pi$ the path of P and the line $y = -x$ intersect.

Question 4 – Three circles

Take a new answer sheet for every question!

Given is the circle c_1 with equation $(x - 4)^2 + y^2 = 4$.

Lines ℓ and m are the tangent lines to circle c_1 that pass through the origin $O(0,0)$.

6pt a Compute algebraically the angle between lines ℓ and m .

Circle c_2 has centre $M(12,6)$ and has no intersections with the x-axis.

The distance between circle c_1 and circle c_2 is 5.

5pt b Compute exactly an equation for circle c_2 .

Circle c_3 passes through the centres of circles c_1 and c_2 .

The centre of circle c_3 is on the line with equation $y = x + 2$.

6pt c Compute exactly the radius of circle c_3 .

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$