**CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE**

# Entrance Exam Wiskunde B

Date: 19 December 2023
Time: 13.30 – 16.30
Questions: 4

**Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.**

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

| Points that can be scored for each item: | | | | |
|---|---|---|---|---|
| Question | 1 | 2 | 3 | 4 |
| a<br>b<br>c<br>d | 4<br>5<br>5<br>7 | 4<br>5<br>8<br>8 | 5<br>3<br>5<br>6 | 6<br>6<br>4 |
| Total | 21 | 25 | 19 | 16 |
| Grade $= \frac{\text{total points scored}}{9} + 1$<br>You will pass the exam if your grade is at least 5.5 . | | | | |

## Question 1 – Mixed functions

*Take a new answer sheet for every question!*

For each positive value of $p$, the functions $f_p$ and $F_p$ are given by

$$f_p(x) = \ln(px) \quad \text{and} \quad F_p(x) = x \cdot \big(f_p(x) - 1\big)$$

The function $F_p$ is an antiderivative of $f_p$ for each positive value of $p$.

4pt a Show that this is true.

For each positive value of $q$, the function $g_q$ is given by

$$g_q(x) = x^q - \ln(x)$$

There is one value of $q$ for which the graph of $g_q$ passes through the point $A\left(\sqrt{e}, 3\frac{1}{2}\right)$.

5pt b Compute exactly this value of $q$ and write the answer in the form $q = \ln(a)$, with $a$ a real number.

5pt c Compute exactly an equation for the tangent line to the graph of the function $g_4(x) = x^4 - \ln(x)$ in the point on this graph at which $x = \frac{1}{2}\sqrt{2}$.

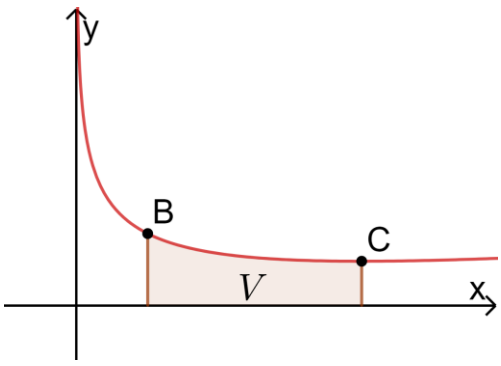
In the figure below, the graph is shown of the function $g_{\frac{1}{2}}(x) = \sqrt{x} - \ln(x)$.

Point $B$ is the point on the graph of $g_{\frac{1}{2}}$ with $x_B = 1$.

Point $C$ is the point on the graph of $g_{\frac{1}{2}}$ where this function has a minimum.

$V$ is the region enclosed by the graph of $g_{\frac{1}{2}}$, the $x$-axis and the vertical lines $x = x_B$ and $x = x_C$.

7pt d Compute exactly the area of region $V$.
*Hint: use the result of question a with $p = 1$.*



## Question 2 – A family of rational functions and 3 circles

*Take a new answer sheet for every question!*

For each value of $p$, the function $f_p$ is given by

$$f_p(x) = \frac{2x^2 + 9x + p}{x + 3}$$

4pt a Compute exactly the value(s) of $p$ for which the graph of $f_p$ has one or more asymptotes.

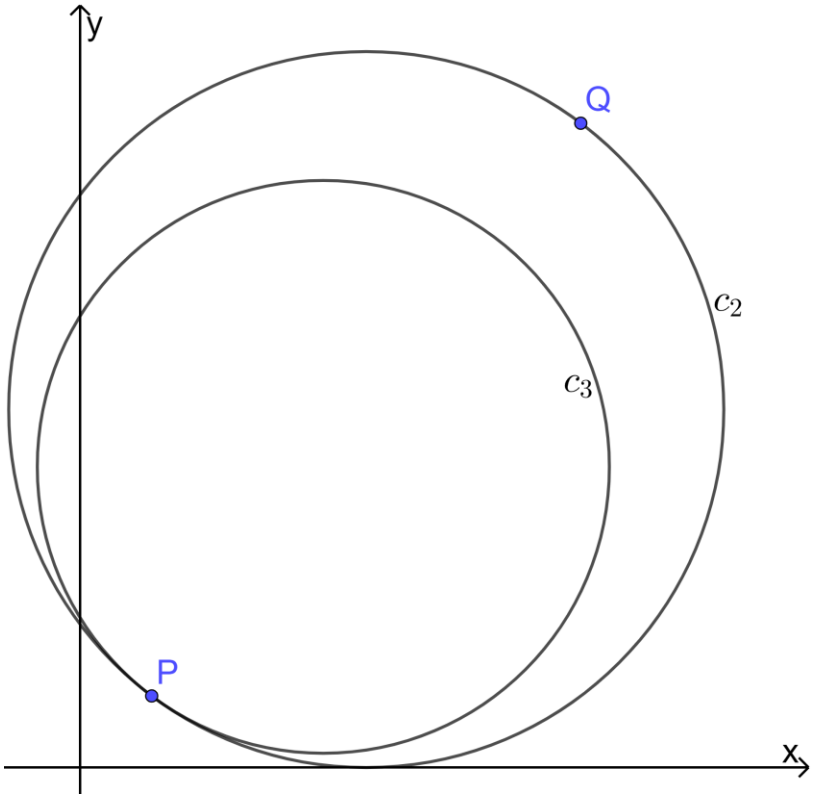
5pt b Compute exactly the value(s) of $p$ for which the graph of $f_p$ has two intersection points with the horizontal line $y = 1$.

Circle $c_1$ touches the graph of $f_{25}(x) = \frac{2x^2+9x+25}{x+3}$ in point $A(1,9)$.
The centre $M$ of $c_1$ is on the line with equation $y = 2x + 1$.

8pt c Compute exactly an equation for circle $c_1$.

In the figure below, the points $P(1,1)$ and $Q(7,9)$ and the circles $c_2$ and $c_3$ are shown. $PQ$ is a diameter of circle $c_2$. Circle $c_3$ touches circle $c_2$ in point $P$. The area of circle $c_3$ is $64\%$ of the area of circle $c_2$.

8pt d Compute exactly an equation for circle $c_3$.



## Question 3 – Three trigonometric functions

*Take a new answer sheet for every question!*

The function $f$ is given by $f(x) = \cos\left(\frac{1}{4}x\right) + \sin\left(\frac{1}{4}x\right)$.

The function $g$ is given by $g(x) = 1 + \sin\left(\frac{1}{2}x\right)$.

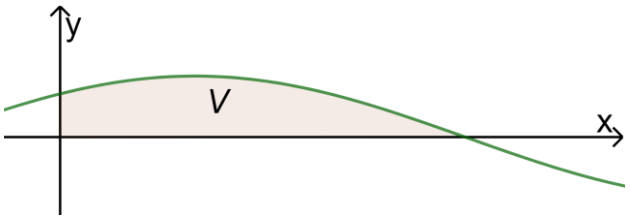
The smallest value of $x$ in the interval $0 \le x \le 16\pi$ for which $f(x) = 0$ is $x = 3\pi$.

5pt a Compute exactly all solutions of the equation $f(x) = 0$ in the interval $0 \le x \le 16\pi$ and thus show that $x = 3\pi$ is indeed the smallest value of $x$ in this interval for which $f(x) = 0$.

For all values of $x$, we have $g(x) = \left(f(x)\right)^2$.

3pt b Use an exact computation to show that this is true.

In the figure below, a part of the graph of the function $f$ is shown.
$V$ is the region enclosed by the $x$-axis, the $y$-axis and this part of the graph of $f$.




5pt c Compute exactly the volume of the solid of revolution that is formed by rotating $V$ around the $x$-axis.

For $x \ge 0$, the function $h$ is given by $h(x) = \cos\left(\sqrt{x}\right)$.

Line $\ell$ is the tangent line to the graph of $h$ in the point $A\left(\frac{1}{4}\pi^2, 0\right)$.

Point $B$ is the intersection of line $\ell$ and the $y$-axis.

Triangle $D$ is the triangle with corners $A$, $B$ and $O(0,0)$.

6pt d Compute exactly the area of triangle $D$.

## Question 4 – Four exponential functions

*Take a new answer sheet for every question!*

The function $f$ is given by $f(x) = 4e^{-x^2+4x-4}$ .
The function $g$ is given by $g(x) = 2e^{3x+3} - 3e^{2x+2}$ .

The graph of $f$ has two points of inflection.

6pt a Compute exactly the *x*-coordinates of the two points of inflection of the graph of $f$.

The functions $f$ and $g$ both have one extreme value.
Point $A$ is the point on the graph of $f$ for which $f(x)$ is maximal.
Point $B$ is the point on the graph of $g$ for which $g(x)$ is minimal.

6pt b Use an exact computation to find a vector representation for the straight line through the points $A$ and $B$.

The function $h$ is given by $h(x) = 2^x$ .
The function $k$ is given by $k(x) = 3^x$ .
Line ℓ is the tangent line to the graph of $h$ in the point $(3,8)$.
Line *m* is the tangent line to the graph of $k$ in the point $(2,9)$.

4pt c Compute algebraically the angle between line ℓ and line *m*.
Give your answer in degrees, rounded to one digit behind the decimal point.

*End of the exam.*

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

*Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.*

*What should **not** be in the folder:*
*- empty sheets, please leave them on your table;*
*- sheets with only your name on it, please take them with you;*
*- scrap paper;*
*- these questions.*

*This is the only way we can ensure a smooth correction of your exam work.*

*Remain seated until one of the invigilators collects your folder (or calls you).*

## Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2 \sin(t) \cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$