

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 25 July 2023
Time: 12.30 – 15.30
Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:				
Question	1	2	3	4
a	8	7	6	4
b	6	8	6	3
c	6	5	7	5
d		5		5
Total	20	25	19	17
Grade = $\frac{\text{total points scored}}{9} + 1$				
You will pass the exam if your grade is at least 5.5 .				

Question 1 – Rational functions

Take a new answer sheet for every question!

The function f is given by

$$f(x) = \frac{x^2 + 4x + 4}{x + 4}$$

Points A and B are the points on the graph of f where the tangent line to this graph is horizontal. Point C is the intersection of the graph of f and the y -axis.

Line ℓ is the straight line through the points A and B .

Line m is the tangent line to the graph of f at point C .

- 8pt a Compute algebraically the angle between line ℓ and line m .
Give your answer in degrees, rounded to one digit behind the decimal point.

The function g is given by

$$g(x) = \frac{1}{x^2 + x - 2}$$

- 6pt b Compute exactly the values of p for which the horizontal line $y = p$ has two intersection points with the graph of g .

The function h is given by

$$h(x) = \frac{(x^2 - 4)(x + 1)}{x^2 - 3x + 2}$$

The graph of h has two asymptotes.

- 6pt c Compute exactly the equations of these two asymptotes.

Question 2 – Exponential and logarithmic functions and a circle

Take a new answer sheet for every question!

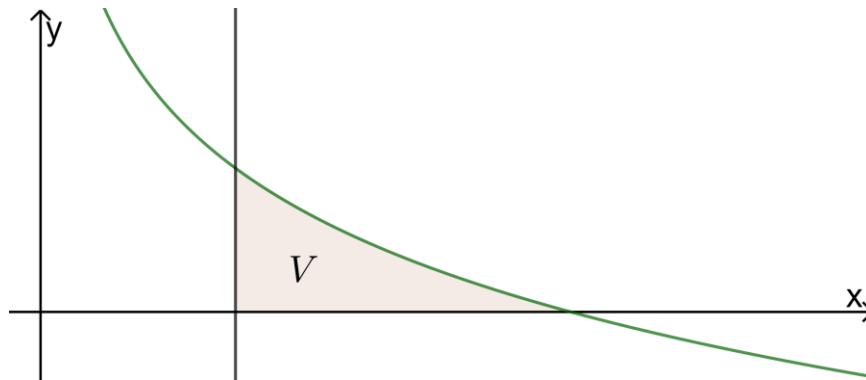
The function f is given by $f(x) = e^{1-x^2}$.

Circle c touches the graph of f at the points $A(-1,1)$ and $B(1,1)$.

7pt a Compute exactly an equation for circle c .

In the figure below, the graph is shown of the function $g(x) = 4 - 2 \ln(x)$.

V is the region enclosed by the graph of g , the x -axis and the vertical line $x = e$.



8pt b Compute exactly the volume of the solid of revolution that is formed by rotating V around the y -axis.

The functions h and k are given by $h(x) = 2 \ln(x)$

and $k(x) = \ln(x - 2) + \ln(2x + 3)$.

5pt c Compute exactly the value(s) of x for which $h(x) = k(x)$.

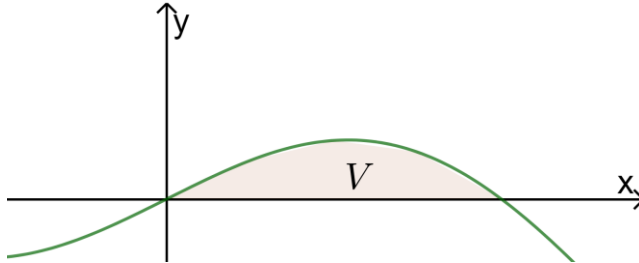
For each real value of p , the function l_p is given by $l_p(x) = \ln(x^2 + p + 2)$.

5pt d Compute exactly the values of p for which the graph of the function l_p has one or more inflection points.

Question 3 – Three trigonometric functions

Take a new answer sheet for every question!

In the figure below, a part of the graph of the function $f(x) = \sin(x) - \sin\left(\frac{1}{2}x\right)$ is shown. V is the region enclosed by this part of the graph and the x -axis.



6pt a Compute exactly the area of region V .

For x in the interval $0 \leq x \leq 2\pi$, the function g is given by

$$g(x) = \sin(x) + \cos(2x)$$

6pt b Compute exactly the x -coordinates of the intersections of the graph of g and the x -axis in the given interval.

The function h is given by $h(x) = \sin(x) \cdot \cos(2x)$.

The graph of h has four horizontal tangent lines.

7pt c Use an exact computation to find an equation for each of these four horizontal tangent lines.

Question 4 – Lines, circles and triangles

Take a new answer sheet for every question!

Given are the points $A(-6,1)$ and $B(1,8)$.

Line ℓ is parallel to the line through points A and B and passes through the point on the positive x -axis which has a distance of 10 to point B .

4pt a Use an exact computation to find a vector representation for line ℓ .

Circle c_1 is the circle with center $P(-1,3)$ that passes through point A .

Point B is also on circle c_1 .

3pt b Use an exact computation to show this.

There are points D on circle c_1 such that triangle ABD is a right-angled triangle.

5pt c Compute exactly the coordinates of these points.

Circle c_2 with centre M also passes through the points A and B .

The area of triangle ABM is 14.

5pt d Compute exactly the area of circle c_2 .

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$