

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 21 April 2023
Time: 13.30 – 16.00 (150 minutes)
Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Note that because of the limited availability of the exam hall, the time of this exam is reduced to 2.5 hours (150 minutes). Of course, the number of items per question is reduced too.

Points that can be scored for each item:				
Question	1	2	3	4
a	6	7	3	7
b	6	4	7	7
c	6	6	5	4
d			4	
Total	18	17	19	18
Grade = $\frac{\text{total points scored}}{8} + 1$				
You will pass the exam if your grade is at least 5.5 .				

Question 1 – Exponential and logarithmic functions

Take a new answer sheet for every question!

The function f is given by $f(x) = 3e^{2x}$.

6pt a Compute exactly the value of a for which

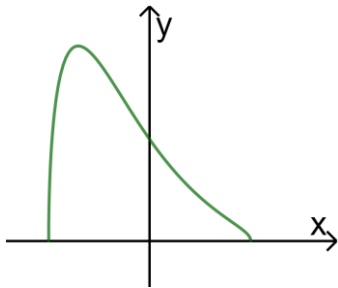
$$\int_0^a f(x) \, dx = 72$$

The function g is given by $g(x) = \ln(x^2 + 3)$.

The graph of this function has two inflection points.

6pt b Compute exactly the x -coordinates of these two inflection points.

In the figure below, the graph is shown of the function $h(x) = e^{-x} \cdot \sqrt{2 - x^2}$.



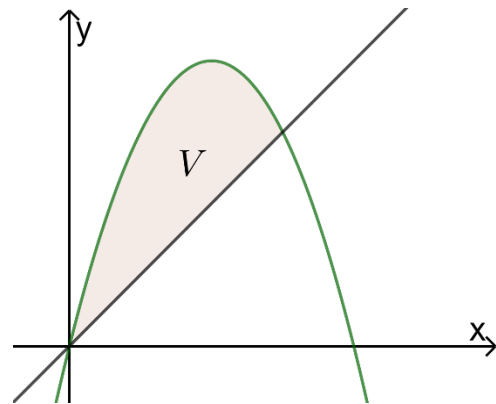
6pt c Compute exactly the maximal value of $h(x)$.

Question 2 – Polynomial and rational functions

Take a new answer sheet for every question!

In the figure on the right, the graph of the function $f(x) = -x^2 + 4x$ and the line l with equation $y = x$ are shown.

V is the bounded region enclosed by the graph of f and line l .



- 7pt a Compute exactly the volume of the solid of revolution that is formed by rotating V around the x -axis.

Also given is the family of functions $g_a(x) = x^2 + a$.

- 4pt b Compute exactly the value(s) of a for which the graphs of f and g_a are touching.

The function h is given by

$$h(x) = \frac{2x^2 + 5x - 3}{x^2 - 9}$$

The graph of h has one perforation (that is a removable discontinuity).

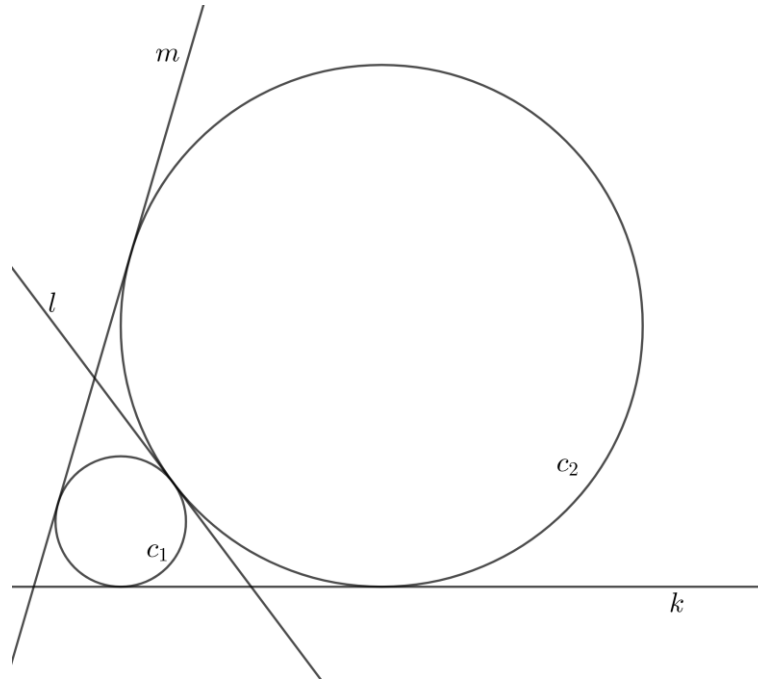
- 6pt c Compute exactly the coordinates of the point where the graph of h has this perforation.

Question 3 – Two circles and three tangent lines

Take a new answer sheet for every question!

In the figure below, the circle c_1 with equation $(x - 4)^2 + (y - 3)^2 = 9$ and the circle c_2 with equation $(x - 16)^2 + (y - 12)^2 = 144$ are shown.

These circles have three tangent lines in common, k , l and m .



One of these common tangent lines, line k in the figure, is the x -axis.

- 3pt a Show that the x -axis is indeed a tangent line to both circles.
- 7pt b Find a vector representation for line l .
- 5pt c Compute exactly the sine of the angle between line m and line k .

Points $A(4,0)$ and $B(4,6)$ are on circle c_1 .

Point C is a point on circle c_1 for which $\angle ABC = 33^\circ$.

- 4pt d Compute exactly $\angle BAC$.

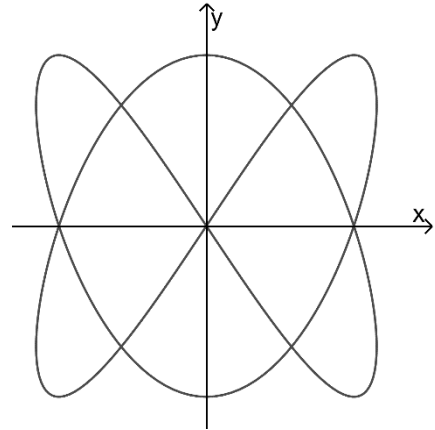
Question 4 – Two moving points

Take a new answer sheet for every question!

The movement of a point P is given by the parametric equations

$$\begin{cases} x(t) = \sin(2t) \\ y(t) = \cos(3t) \end{cases} \quad \text{with } -\pi \leq t \leq \pi.$$

In the figure on the right, the path of point P is shown.



- 7pt a Compute exactly the values of t on the interval $-\pi \leq t \leq \pi$ for which the path of P intersects the line with equation $y = x$.
- 7pt b Compute algebraically the angle at which the path of P intersects itself in the origin $O(0,0)$.

The movement of a point Q is given by the parametric equations

$$\begin{cases} x(t) = \cos(t) + 1 \\ y(t) = 4 \cos(t) - 2 \sin^2(t) \end{cases} \quad \text{with } -\pi \leq t \leq \pi.$$

The path of point Q is part of a parabola with an equation of the form $y = ax^2 + bx + c$.

- 4pt c Use an exact computation to find the values of a , b and c .

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$