

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 16 December 2022

Time: 13.30 – 16.30

Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:				
Question	1	2	3	4
a	6	6	6	6
b	5	6	3	5
c	6	6	6	6
d	3		7	4
Total	20	18	22	21
Grade = $\frac{\text{total points scored}}{9} + 1$				
You will pass the exam if your grade is at least 5.5 .				

Question 1 – Functions with square roots

Take a new answer sheet for every question!

The function f is given by $f(x) = \sqrt{x+4} \cdot \sqrt{12-x^2}$.

The line ℓ is given by the equation $y = 4\sqrt{3}$.

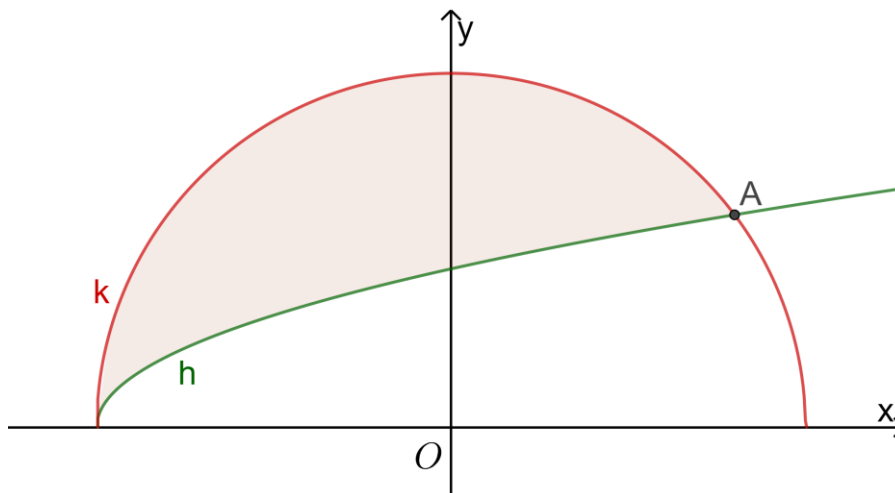
- 6pt a Compute exactly the coordinates of the intersection(s) of line ℓ and the graph of f .

The function g is given by $g(x) = \sqrt{x+2} \cdot \sqrt{7-x^2}$.

This function has one maximum.

- 5pt b Compute exactly the coordinates of the point on the graph of g where this function has its maximum.

In the figure below, the graphs are shown of the functions $h(x) = \sqrt{x+5}$ and $k(x) = \sqrt{25-x^2}$. Point $A(4,3)$ is an intersection of these graphs.



V is the bounded region enclosed by the graphs of h and k (shaded in the figure above).

- 6pt c Compute exactly the volume of the solid of revolution that is formed by rotating V around the x -axis.

For each value of a , the function q_a is given by

$$q_a(x) = \frac{\sqrt{16-x^2}}{\sqrt{x+a}}$$

- 3pt d Determine the value(s) of a for which the graph of q_a has an asymptote.

Question 2 – Exponential and logarithmic functions

Take a new answer sheet for every question!

In the figure on the right, the graph is shown of the function $f(x) = 9 - 4e^{2x}$.

V is the region enclosed by the graph of f , the x -axis and the y -axis (shaded in the figure).

6pt a Compute exactly the area of region V .

Furthermore the points $A(0, 5)$ and $B(1, 8)$ are given. Line ℓ is the tangent line to the graph of f in point $A(0, 5)$. Line m is the line through the origin $O(0, 0)$ and point $B(1, 8)$.

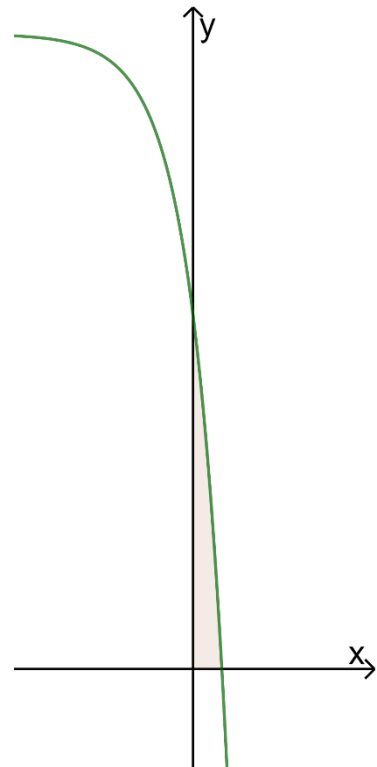
6pt b Compute algebraically the angle between lines ℓ and m .
Give your answer in degrees.

Finally, the function g is given by

$$g(x) = \ln(x^2 + 6x + 18).$$

The graph of this function has two inflection points.

6pt c Compute exactly the x -coordinates of these points.



Question 3 – Two trigonometric functions and a tangent line

Take a new answer sheet for every question!

For x in the interval $0 \leq x \leq 2\pi$, the function f is given by

$$f(x) = 4 \sin(x) \cos(x)$$

Point A with x -coordinate $x_A = \frac{1}{6}\pi$ is on the graph of f .

- 6pt a Use an exact computation to find an equation for the tangent line to the graph of f in point A .

The function $F(x) = -\cos(2x)$ is an antiderivative of f .

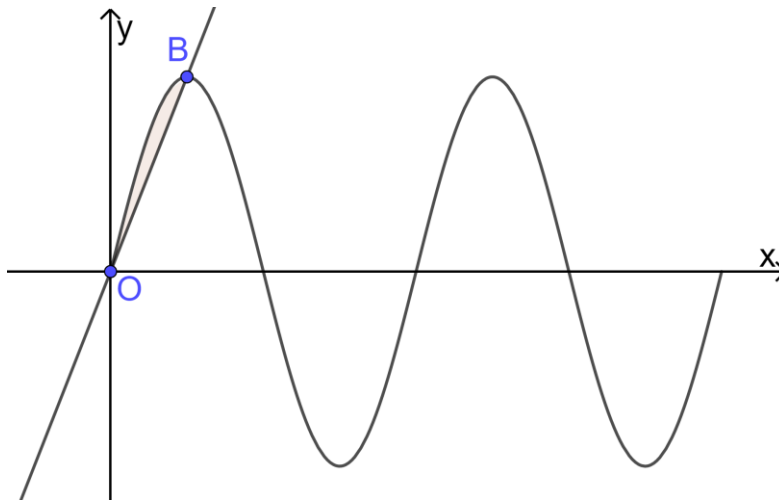
- 3pt b Use an exact computation to show this.

Point B with x -coordinate $x_B = \frac{1}{4}\pi$ is also on the graph of f .

ℓ is the line through the origin $O(0, 0)$ and point B .

V is the region enclosed by the graph of f and line ℓ .

See the figure below.



- 6pt c Compute exactly the area of region V .

For x in the interval $0 \leq x \leq 2\pi$, the function g is given by

$$g(x) = \sin(\pi \cdot \cos(x))$$

- 7pt d Compute exactly the coordinates of the points on the graph of g in the given interval where the function g has an extreme value (minimum or maximum).

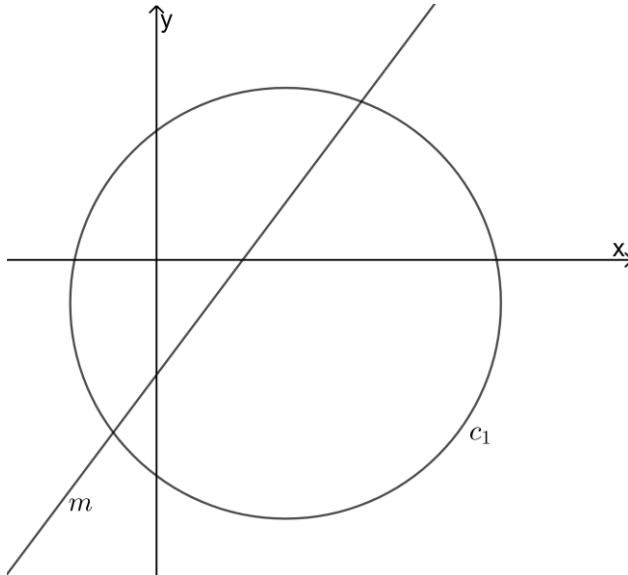
Question 4 – Circles, lines and a triangle

Take a new answer sheet for every question!

Circle c_1 is given by the equation $(x - 3)^2 + (y + 1)^2 = 25$.

Line m is given by the vector representation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

See the figure below.



Two lines that are parallel to the x -axis and having a distance of 2 to circle c_1 intersect line m in points A and B respectively.

6pt a Compute exactly the distance between points A and B .

There are two points on circle c_1 at which the tangent line to c_1 is parallel to line m .

5pt b Compute exactly the coordinates of these two points.

A line with equation $y = p$ intersects the given line m in a point N such that N is the centre of a circle c_2 with radius $8\sqrt{2}$ which passes through the origin $O(0, 0)$.

6pt c Compute exactly the possible values of p .

Points $P(0, 3)$ and $Q(6, -5)$ are on the circle c_1 with equation $(x - 3)^2 + (y + 1)^2 = 25$.

Point R is a point on circle c_1 such that $\angle PQR = 30^\circ$.

4pt d Compute exactly for the triangle PQR :

- 1) $d(P, Q)$, the length of side PQ ;
- 2) $\angle QPR$, the angle between sides PQ and PR .

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).