

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 23 July 2022
Time: 13.30 – 16.30
Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

| Points that can be scored for each item: | | | | |
|--|----|----|----|----|
| Question | 1 | 2 | 3 | 4 |
| a | 5 | 6 | 3 | 5 |
| b | 5 | 7 | 6 | 8 |
| c | 8 | 5 | 6 | 6 |
| d | | 5 | 6 | |
| Total | 18 | 23 | 21 | 19 |
| Grade = $\frac{\text{total points scored}}{9} + 1$ | | | | |
| You will pass the exam if your grade is at least 5.5 . | | | | |

Question 1 – A rational function and a family of parabola

Take a new answer sheet for every question!

The function f is given by $f(x) = \frac{8}{x^2}$.

The family of functions g_a is given by $g_a(x) = a - \frac{1}{2}x^2$.

5pt a Compute exactly the coordinates of the intersection points of the graph of f and the graph of $g_5(x) = 5 - \frac{1}{2}x^2$.

For each $p > 0$, the vertical line ℓ_p with equation $x = p$ is given.

F_p is the intersection point of ℓ_p and the graph of f .

G_p is the intersection point of ℓ_p and the graph of $g_1(x) = 1 - \frac{1}{2}x^2$.

5pt b Compute exactly the value of p for which the distance between F_p and G_p is minimal.

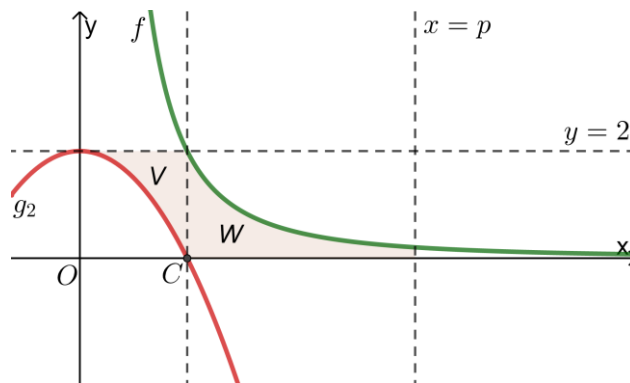
In the figure below, the graphs are shown of the function f and of the function

$$g_2(x) = 2 - \frac{1}{2}x^2.$$

C is the intersection point of the graph of g_2 and the positive x -axis.

V is the region enclosed by the graph of g_2 , the horizontal line $y = 2$ and the vertical line through C .

W is the region enclosed by the graph of f , the vertical line through C , the x -axis and the line ℓ_p with equation $x = p$ with $p > x_C$.



There is a value of p for which the area of V is precisely half of the area of W .

8pt c Compute this value of p exactly.

Question 2 – Exponential and logarithmic functions

Take a new answer sheet for every question!

The function f is given by

$$f(x) = \frac{e^x}{e^x + 2}$$

The graph of f has one point of inflexion.

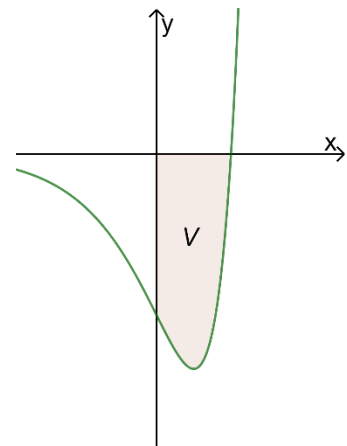
6pt a Compute exactly the coordinates of this point.

In the figure on the right, the graph is shown of the function

$$g(x) = e^x \cdot (e^x - 4)$$

V is the region enclosed by the x -axis, the y -axis and the graph of g .

7pt b Compute exactly the volume of the solid of revolution that is formed by rotating V around the x -axis.



The function h is given by

$$h(x) = \ln(x + 2) + \frac{1}{3}\ln(x^3)$$

5pt c Compute exactly the coordinates of the intersection point(s) of the graph of h and the line with equation $y = \ln(8)$.

For each real value of p , the function k_p is given by

$$k_p(x) = \ln(px^2 - px + 1)$$

5pt d Compute exactly the value of p for which the tangent line to the graph of k_p in $O(0,0)$ passes through point $A(1,2)$.

Question 3 – A trigonometric function and a triangle

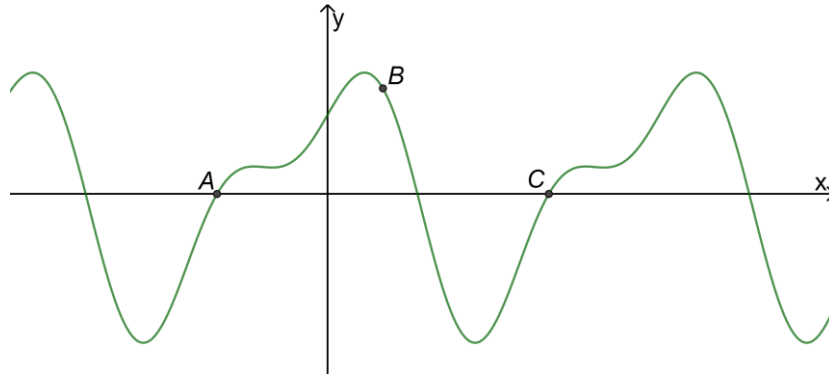
Take a new answer sheet for every question!

In the figure below, the graph is shown of the function

$$f(x) = 2 \cos(x) + \sin\left(2x - \frac{1}{6}\pi\right)$$

$A\left(-\frac{2}{3}\pi, 0\right)$, $B\left(\frac{1}{3}\pi, 2\right)$ and $C\left(\frac{4}{3}\pi, 0\right)$ are all points on the graph of f .

You do not have to show this.



The derivative function of f can be written as

$$f'(x) = -2 \sin(x) + 2 \sin\left(2x + \frac{1}{3}\pi\right)$$

3pt a Use an exact computation to show this.

6pt b Compute exactly the x -coordinates of the points on the graph of f in the interval $-2\pi \leq x \leq 2\pi$ at which the tangent line to this graph is horizontal.

Line m is the tangent line to the graph of f at point A .

Line n is the tangent line to the graph of f at point B .

Point D is the intersection point of m and n .

Point E is the intersection point of n and the x -axis.

6pt c Use an exact computation to show that triangle ADE is equilateral.

The graph of f and the x -axis enclose two types of regions: the ones below the x -axis and the ones above the x -axis.

6pt d Use an exact computation to investigate whether these types of regions are equal in area.

Question 4 – A rational function and two circles

Take a new answer sheet for every question!

The function f is given by

$$f(x) = \frac{8x - 12}{6x^2 - 17x + 12}$$

The graph of this function has two asymptotes.

- 5pt a Use an exact computation to find an equation for each of these asymptotes.

Circle c_1 touches the graph of f at the intersection point of this graph and the y -axis. The centre of circle c_1 lies on the line with parametric representation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- 8pt b Use an exact computation to find an equation for circle c_1 .

Circle c_2 is given by the equation

$$x^2 - 8x + y^2 - 6y = 0$$

A and B are two points on this circle. The tangent lines to the circle in these points intersect at an angle of 30° .

- 6pt c Compute algebraically the distance between points A and B .
Give your answer rounded to two digits behind the decimal point.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$