

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 22 April 2022
Time: 13.30 – 16.30
Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

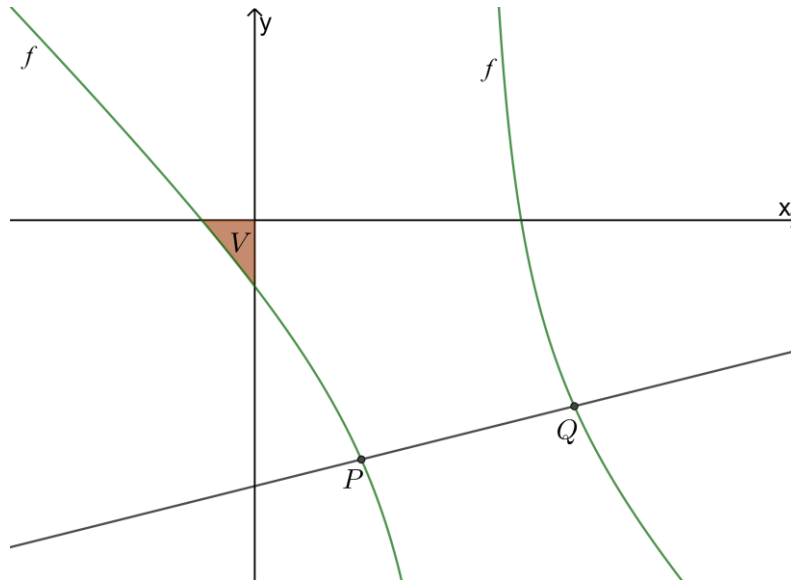
Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:				
Question	1	2	3	4
a	6	6	6	6
b	3	6	7	6
c	7	5	7	7
d	5	4		
Total	21	21	20	19
Grade = $\frac{\text{total points scored}}{9} + 1$				
You will pass the exam if your grade is at least 5.5 .				

Question 1 – A rational function

Take a new answer sheet for every question!

In the figure below, the graph is shown of the function $f(x) = \frac{x^2 - 4x - 5}{4 - x}$.



P and Q are the points on the graph of f with respectively $x_P = 2$ and $x_Q = 6$.

ℓ is the straight line that intersects the graph of f in the points P and Q .

- 6pt a Use an exact computation to investigate whether line ℓ intersects the graph of f perpendicularly in point P .

The function rule of f can be rewritten into $f(x) = -x + \frac{5}{x-4}$.

- 3pt b Show this.

V is the region enclosed by the graph of f , the x -axis and the y -axis. This region is coloured in the figure above.

- 7pt c Compute exactly the area of region V .

Furthermore given is the family of functions $g_a(x) = \frac{x^2 - 4x - 5}{a - x}$.

For $a = 4$ this is precisely the function f given above. The graph of this function has an oblique asymptote and a vertical asymptote. However, there are values of a for which the graph of g_a has no asymptotes.

- 5pt d Compute these values of a and draw the graph of the function g_a for the smallest of these values of a .

Question 2 – The path of a moving point

Take a new answer sheet for every question!

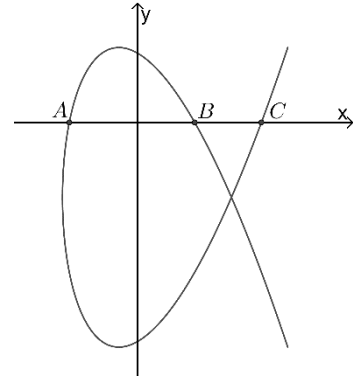
The path of a moving point $P(x, y)$ is for any value of p and q given by

$$\begin{cases} x(t) = 2 - 3 \sin^2(t) \\ y(t) = p \cdot \cos(qt) - 1 \end{cases} \text{ with } 0 \leq t \leq 2\pi.$$

For $p = 2$ and $q = 3$ the path of point P is given by

$$\begin{cases} x(t) = 2 - 3 \sin^2(t) \\ y(t) = 2 \cos(3t) - 1 \end{cases}$$

This path intersects the x -axis in three points, A , B and C , see the figure on the right.



- 6pt a For each of these points, compute exactly the corresponding value of t in the interval $0 \leq t \leq 2\pi$.

The path of point P has points where the tangent line to the path runs parallel to the x -axis.

- 6pt b Compute exactly the coordinates of these points.

We now take $p = \frac{3}{2}$ and $q = 1$.

The path of P is then given by

$$\begin{cases} x(t) = 2 - 3 \sin^2(t) \\ y(t) = \frac{3}{2} \cos(t) - 1 \end{cases} \text{ with } 0 \leq t \leq 2\pi.$$

- 5pt c Compute exactly for which values of t in the interval $0 \leq t \leq 2\pi$ the path of P and the line $y = x$ intersect.

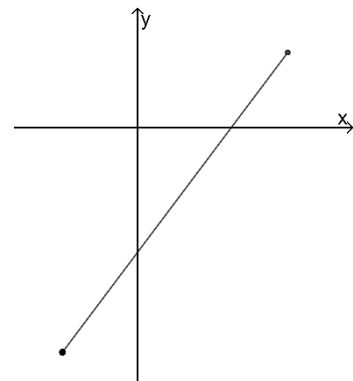
Finally, we take $p = 2$ and $q = 2$.

The path of P is then given by

$$\begin{cases} x(t) = 2 - 3 \sin^2(t) \\ y(t) = 2 \cos(2t) - 1 \end{cases} \text{ with } 0 \leq t \leq 2\pi.$$

In this case, the path of P is the line segment shown in the figure to the right.

The line m of which this line segment is a part has the equation $y = \frac{4}{3}x - 1\frac{2}{3}$.



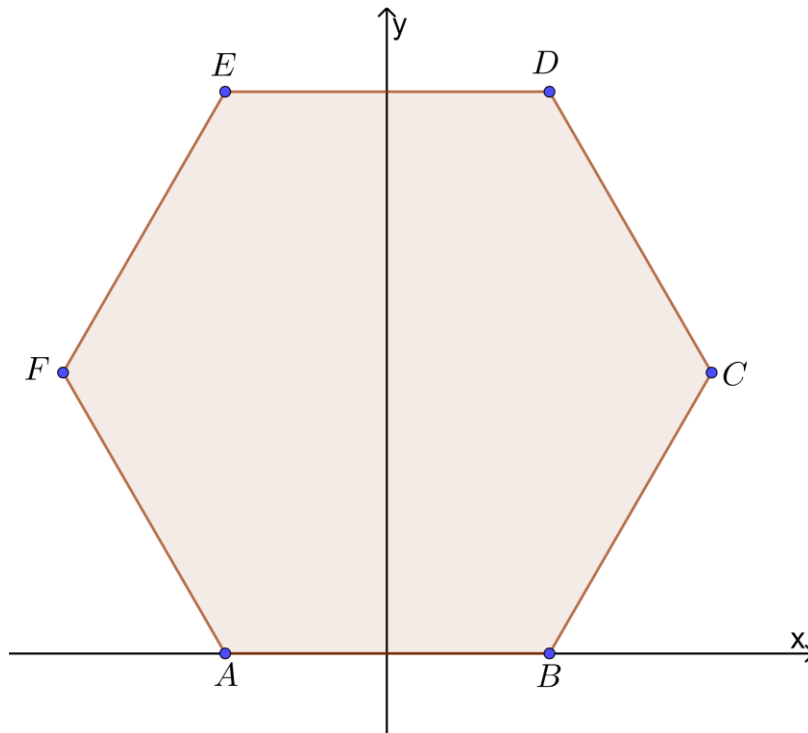
- 4pt d Show that all points of the path of P are on line m .

Question 3 – A regular hexagon and a circle

Take a new answer sheet for every question!

A regular hexagon is a hexagon of which all sides are equal in length and all angles are equal to 120° .

The figure below, points $A(-3,0)$ and $B(3,0)$ are shown, that together with the points C, D, E and F form a regular hexagon.



Point C has coordinates $(6, 3\sqrt{3})$.

6pt a Use an exact computation to show this.

Point R is the midpoint of side EF .

7pt b Compute algebraically the angle at which the line through R and B intersects the y -axis.
Give your answer rounded to whole degrees.

For each value of p the line ℓ_p is given by the vector representation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ p \end{pmatrix}$$

Also given is the circle c with centre B that passes point A .

7pt c Compute exactly the values of p for which ℓ_p is a tangent line to circle c .

Question 4 – Two logarithmic functions

Take a new answer sheet for every question!

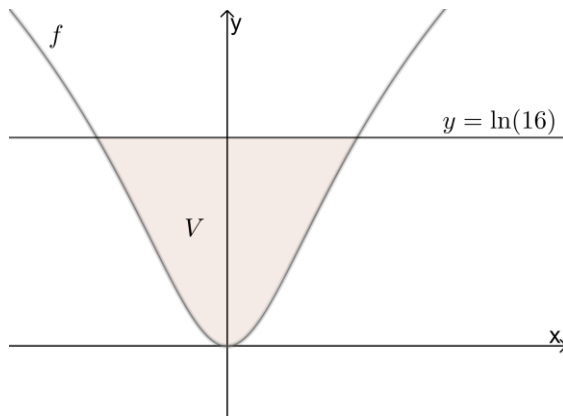
Given are the functions $f(x) = 2 \ln(x^2 + 1)$ and $g(x) = \ln\left(\frac{9}{2}x^2\right)$.

- 6pt a Compute exactly the coordinates of the intersections of the graphs of f and g .

The graph of f has two points of inflexion.

- 6pt b Compute exactly the x-coordinates of these points of inflexion.

In the figure below, V is the region enclosed by the graph of f and the horizontal line $y = \ln(16)$.



- 7pt c Compute exactly the volume of the solid of revolution that is formed by rotating V around the y -axis.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$