

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 18 December 2021

Time: 13.30 – 16.30

Questions: 5

Answer sheets: 1 (for question 4b)

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:					
Question	1	2	3	4	5
a	7	3	4	6	5
b	8	3	4	6	5
c		8	6		6
d			3		
e			7		
Total	15	14	24	12	16
Grade = $\frac{\text{total points scored}}{9} + 1$					
You will pass the exam if your grade is at least 5.5 .					

Question 1 – A rational function

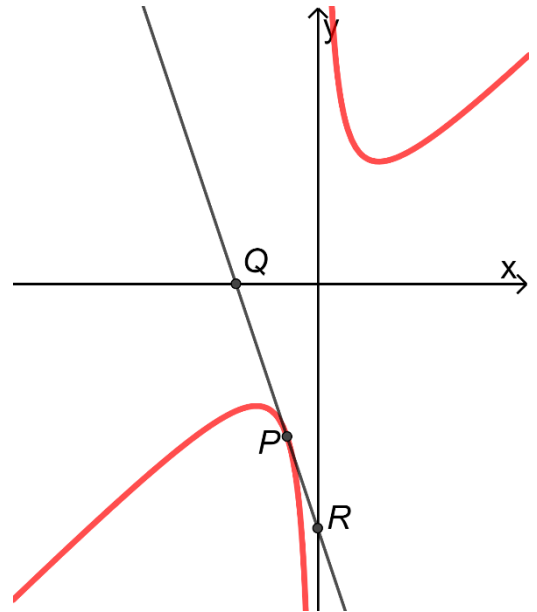
Take a new answer sheet for every question!

The function f is given by

$$f(x) = x + \frac{1}{x}$$

The point $P\left(-\frac{1}{2}, -2\frac{1}{2}\right)$ is on the graph of f .

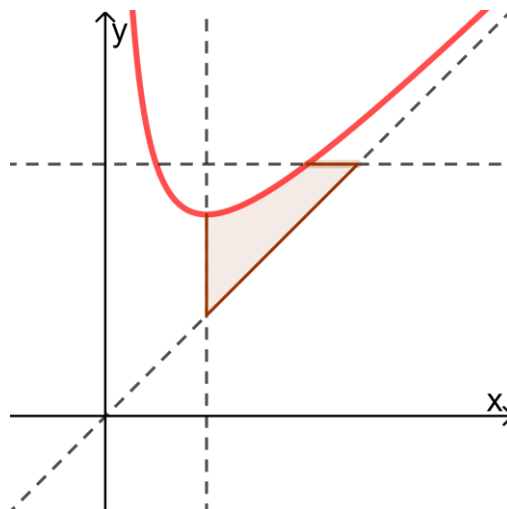
The tangent line to the graph of f at point P intersects the x -axis in point Q and intersects the y -axis in point R . See the figure on the right.



- 7pt a Compute exactly the distance between the points Q and R .

In the figure below, the region enclosed by the graph of f , the line $y = 2\frac{1}{2}$, the line $y = x$ and the line $x = 1$ is coloured.

- 8pt b Compute exactly the area of this region.



Question 2 – Rational functions and a circle

Take a new answer sheet for every question!

The family of functions g_p is given by

$$g_p(x) = \frac{x^2 + px + p}{x}$$

- 3pt a Compute exactly the values of p for which the graph of g_p has two intersections with the x -axis.

In the figure on the right, the graph is shown of the function

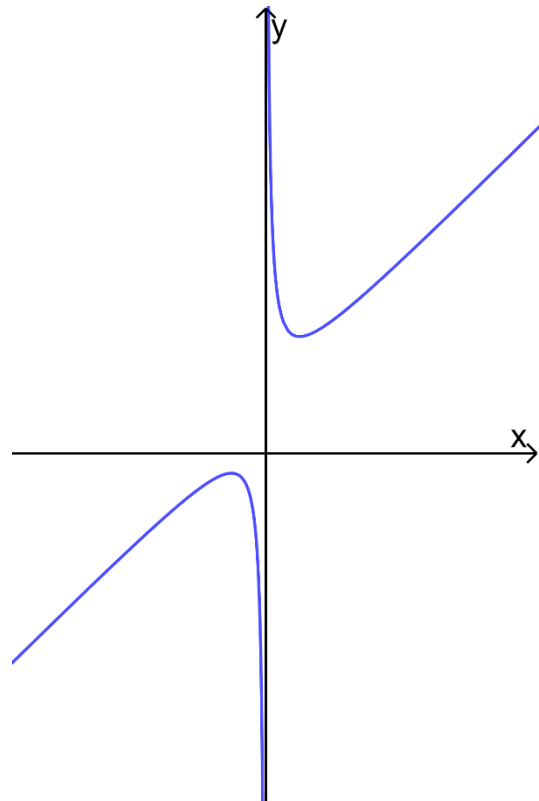
$$g_2(x) = \frac{x^2 + 2x + 2}{x}$$

The graph of g_2 has two asymptotes.

- 3pt b Find the equations for these two asymptotes.

The circle c touches the graph of g_2 in point $P(2,5)$. The centre of c is on the y -axis.

- 8pt c Compute exactly an equation for this circle.



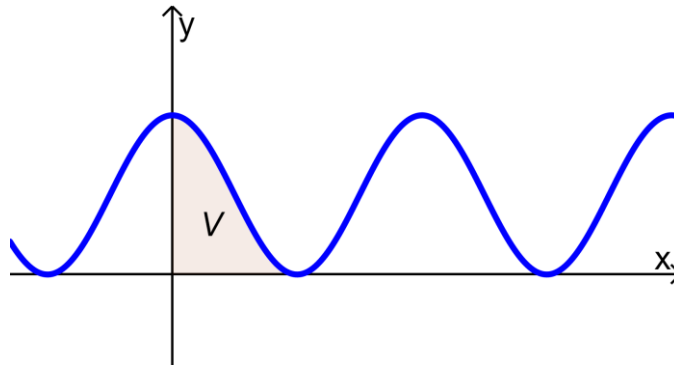
Question 3 – Functions with waving graphs

Take a new answer sheet for every question!

Given is the function $f(x) = \cos^2(2x)$.

In the figure below, the graph of f is shown.

In this figure, a region V is coloured. This is the region to the right of the y -axis enclosed by the graph of f , the x -axis and the y -axis.



The function $F(x) = \frac{1}{2}x + \frac{1}{8}\sin(4x)$ is an antiderivative of f .

4pt a Show that this is true.

4pt b Compute exactly the area of region V .

The graph of f has a point of inflection in $P\left(\frac{1}{8}\pi, \frac{1}{2}\right)$.

6pt c Use the second derivative of f to prove this.

3pt d Compute algebraically the angle between the tangent line to the graph of f in point P and the y -axis. Give your answer rounded to whole degrees.

Also given is the function $g(x) = 4\frac{1}{2}\cos(2x) - 2$.

7pt e Compute exactly the x -coordinates of the intersections of the graphs of f and g on the interval $0 \leq x \leq \pi$.

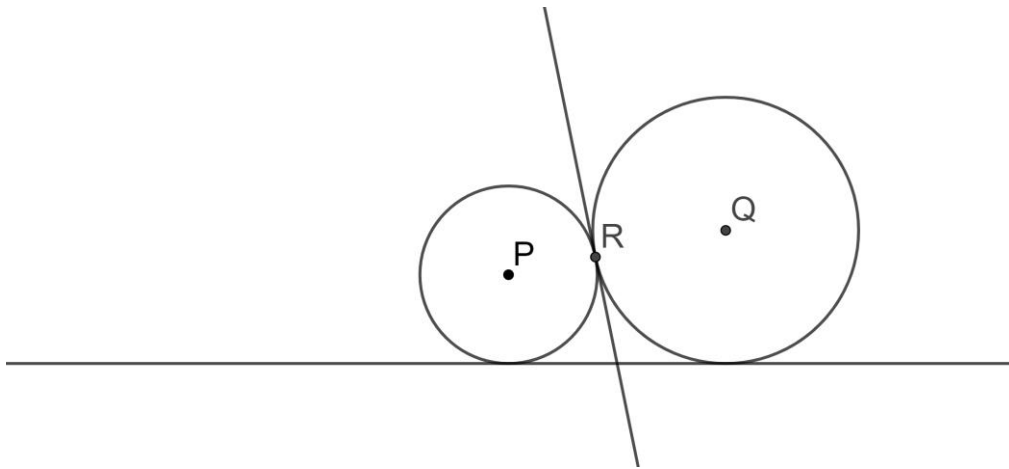
Question 4 – Circles and (tangent) lines

Take a new answer sheet for every question!

Given are the circle c with equation $(x - 15)^2 + (y - 5)^2 = 25$ and the line m with vector representation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

6pt a Compute exactly the distance between the line m and the circle c .

In the figure below, two circles are shown with centres P and Q , which are touching in point R . Also shown are two common tangent lines of these circles.



These circles have a third tangent line in common.

6pt b - Draw this tangent line on the printed answer sheet.
- Show that the line that you have drawn is indeed a tangent line to both circles.

Question 5 – Exponential and logarithmic functions

Take a new answer sheet for every question!

Given are the functions $f(x) = \ln(2x + 1)$ and $g(x) = \frac{1}{2} \ln(x)$.

5pt a Solve the equation $f(x) - g(x) = \ln(3)$ exactly.

Furthermore, for every value of a and b is given the function $h_{ab}(x) = a + e^{bx}$.

5pt b Compute exactly the values of a and b for which the graphs of h_{ab} and f are touching in the origin $O(0,0)$.

V is the region enclosed by the graph of $h_{31}(x) = 3 + e^x$, the y -axis, the x -axis and the vertical line $x = \ln(2)$.

6pt c Compute exactly the volume of the solid of revolution that is formed by rotating V around the x -axis.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$