CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 17 April 2021
Time: 140 minutes (2 hours and 20 minutes)
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Because the time for this exam has been reduced to 140 minutes, the number of items per question has been reduced. Therefore, the total number of points that can be scored is reduced to 72.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c d	5 5 6 4	4 4 3	5 1 4 2	3 5 4	5 5	2 5
Total	20	11	12	12	10	7
Grade $= \frac{\text{total points scored}}{8} + 1$ You will pass the exam if your grade is at least 5.5 .						

Question 1 – Algebraic skills

Take a new answer sheet for every question!

*When you are asked to perform a computation **algebraically**, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed either in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$*.*
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = x^3 + 4x^2 - 8x + 4$.
The function g is given by $g(x) = (2x + 2)^2$.
The line ℓ is given by the equation $y = -5x + 7$.

5pt a Compute algebraically the *x*-coordinates of the intersections of the graphs of f and g .

5pt b Compute algebraically the values of a for which the tangent line to the graph of f at the point $(a, f(a))$ is parallel to line ℓ .

The function h is given by $h(x) = \sqrt{4x^2 + 3}$.

6pt c Compute algebraically the value(s) of x for which $h'(x) = 1$.

The function k is given by $k(x) = x \cdot \mathrm{e}^{-2x+2}$.
Point *P* is the intersection of the graph of k and the vertical line with equation $x = 1$.

4pt d Compute algebraically the slope of the graph of k in point *P* .

Question 2 – Profit on a commodity

Take a new answer sheet for every question!

The profit on a commodity is the difference between the revenue and the cost.
For a certain commodity, the cost C is given by the formula

$$C = \frac{q^2 + 8q + 4}{q + 4}$$

In this formula, C is in thousands of euros and q is the weight of the commodity in tons (1 ton = 1000 kg, so q does not have to be an integer number).

This commodity is produced on demand, so all units that are produced will be sold.

The price of the commodity is 2 euro per kg, so the total revenue for the sale of this commodity is given by

$$R = 2q$$

with R in thousands of euros.

The total profit on this commodity in thousands of euros is therefore given by

$$P = \frac{q^2 - 4}{q + 4}$$

4pt a Show how this formula for P is derived from the formula's for C and R that are given above.

4pt b Show algebraically that the derivative of the profit function is given by

$$\frac{\mathrm{d}P}{\mathrm{d}q} = \frac{q^2 + 8q + 4}{q^2 + 8q + 16}$$

3pt c Using this derivative, show algebraically that the profit function is increasing for all values of q for which it exists.

Question 3 – Noflue

Take a new answer sheet for every question!

The pharmaceutical company Pillfit produces a new medicine with the name Noflu. The company claims that 20% of the patients who take this medicine will be healed from influenza after one day.

In item a you may assume that this claim is true. Give your answers to item a rounded to four decimals.

10 influenza patients take this new medicine.

5pt a Compute the probability that at least two of these 10 patients are cured from influenza after one day.

Pillfit also claims that 50% of patients who use Noflu will be cured from influenza after two days. The Health counsel suspects that this percentage will be lower and decides to test this.

1pt b State the null hypothesis and the alternative hypothesis for this test procedure.

16 randomly selected patients are given Noflu. After two days, 12 of these patients still have influenza.

4pt c Compute the probability that 12 out of 16 patients still have influenza after 2 days if the above claim of Pillfit is true.

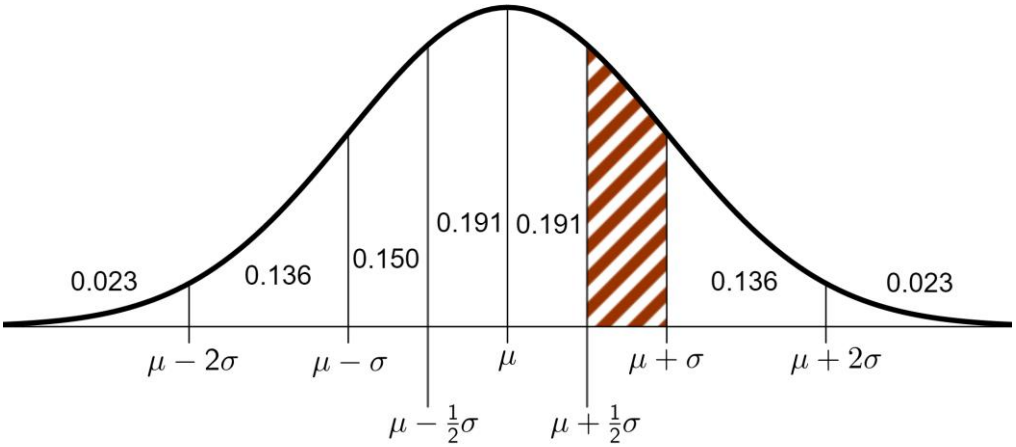
2pt d Can you draw a conclusion for this test procedure based on your answer of item c? If so, state and motivate this conclusion. If not, explain why not.

Question 4 – Headache

Take a new answer sheet for every question!

Pillfit also produces headache pills. The weights of these pills are normally distributed with an average of 500 mg and a standard deviation of 10 mg.

3pt a Use the figure below to compute the percentage of these pills that have a weight between 495 mg and 520 mg.



A normal probability distribution X
The area of the shaded region represents $P\left(\mu + \frac{1}{2}\sigma < X < \mu + \sigma\right) = 0.150$

These pills are sold in boxes of 50. The weights of these boxes are also normally distributed. The average weight of the boxes is 10 g and the standard deviation is 0.5 g. The 50 pills in a box are randomly selected. The total weight of such a package (box + pills) is a random variable *Y*.

5pt b Compute the mean and the standard deviation of *Y*.

Pillfit furthermore sells general pain relief pills for children. These are sold in boxes that contain two orange, three red and four blue pills. Jane's three children all need such a pill, so she randomly selects three pills from such a box.

4pt c Compute the probability that the pills she selects all have the same color.

Question 5 – The R number

Take a new answer sheet for every question!

The basic reproduction number, also known as the R or R0, is the average number of people one person with an infectious disease will likely infect in the future.

For example, an R of 3.5 would mean 100 people with the new coronavirus would likely go on to infect 350 people. Those 350 would in turn transmit it to 1225 people. When the R is above 1, the virus will grow exponentially in a population with no immunity. At 1 it stays steady. Below 1, the virus will gradually infect fewer people, until the epidemic dries up.

Source: https://www.newscientist.com/definition/r-number/

When predicting the number of infected persons, the time in which an infected person can infect other persons also plays an important role. If $R = 2$ and this time is 1 week, the number of newly infected persons will double in 1 week.

5pt a Compute algebraically the time in days in which the number of newly infected persons will double if $R = 1.44$ and the time in which an infected person can infect other persons is 2 days.

The actual R number of a disease will decrease if persons that already had the disease are less likely to catch the disease again and by vaccination. For a certain disease (not corona) this effect is modelled by the formula

$$R = \frac{4}{1 + 2\mathrm{e}^{0.1t}}$$

(t in months, $t = 0$ at the start of the vaccination campaign)

5pt b Compute algebraically the time in days at which according to this formula the actual R number will be equal to 1. Give your answer rounded to whole days (1 month = 30 days).

Question 6 – Up and down

Take a new answer sheet for every question!

A metal sphere is hung from a ceiling by means of a spring. At a certain moment, the sphere is pulled down and then released. From this moment onwards the sphere moves up and down. The distance between the center of the sphere and the ceiling is given by the formula

$$D = 13.5 + 4.0 \sin\left(\frac{1}{3}\pi\left(t + \frac{3}{2}\right)\right)$$

In this formula, t is the time in seconds and D is the distance between the center of the sphere and the ceiling in centimeters.

2pt a Compute the minimal and the maximal distance between the center of the sphere and the ceiling.

At $t = 1$, the distance between the center of the sphere and the ceiling is 15.5 cm.

5pt b Use the period of D to determine the first three times after $t = 1$ at which $D = 15.5$ cm.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\big(g(x)\big)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\big(g(x)\big) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^g\log a + {}^g\log b = {}^g\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a - {}^g\log b = {}^g\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a^p = p \cdot {}^g\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^g\log a = \frac{{}^p\log a}{{}^p\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ with $k = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$ $g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$ $g_r = \mu_T + 2.58\sigma_T$

