CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 15 December 2020
Time: 140 minutes (2 hours and 20 minutes) (Due to corona circumstances this exam has been shortened to 140 minutes)
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Since the time for this exam has been reduced to 140 minutes (instead of 180), the number of items per question is reduced. Therefore, the total number of points that can be scored is reduced to 70 (instead of 90).

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c d	4 5 5 4	2 5 4	4 4	3 3 5 5	2 3 2	5 5
Total	18	11	8	16	7	10
Grade $= \frac{\text{total points scored}}{10} \times \frac{9}{7} + 1$ You will pass the exam if your grade is at least 5.5 .						

Question 1 – Algebraic skills

Take a new answer sheet for every question!

*When you are asked to perform a computation **algebraically**, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed either in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$*.*
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = x^3 - 3x^2$.
The line ℓ is given by the equation $y = 4x$.

4pt a Compute algebraically the coordinates of the intersections of the graph of f and line ℓ .

5pt b Compute algebraically the values of a for which the tangent line to the graph of f at the point $(a, f(a))$ is parallel to line ℓ . Give your answers rounded to two digits behind the decimal point.

The function g is given by $g(x) = \frac{3x^2+12}{2x}$.

5pt c Compute algebraically the values of a for which the tangent line to the graph of g at the point $(a, g(a))$ is horizontal.

The function h is given by $h(x) = x^2 \cdot \mathrm{e}^{-x}$.
Point *P* is the intersection of the graph of h and the vertical line with equation $x = 1$.

4pt d Compute algebraically the slope of the graph of h in point *P* .

Question 2 – Profit (or not)

Take a new answer sheet for every question!

The profit on a commodity is the difference between the revenue and the cost.
For a certain commodity, the cost per unit C for the production Q is given by the formula

$$C = 3 + \frac{11}{Q}$$

In this formula, C is in thousands of euro's and Q is the weight of the commodity in tons (1 ton = 1000 kg, so Q does not have to be an integer number).

This commodity is produced on demand, so all units that are produced will be sold.

The total revenue for the sale of this commodity is given by

$$R = 4 + 10\sqrt{Q}$$

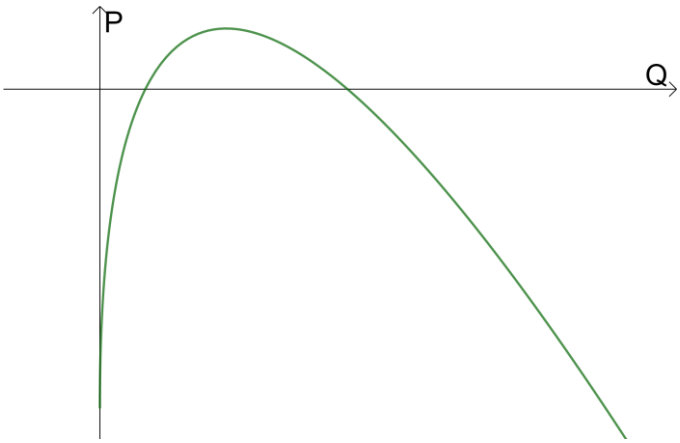
with R in thousands of euro's.

The total profit on this commodity is therefore given by

$$P = -3Q + 10\sqrt{Q} - 7$$

2pt a Show how this formula for P is derived from the formula's for C and R.

In the figure below, the graph that represents the formula for P is shown.



5pt b Compute algebraically the values of Q for which the total profit is positive.

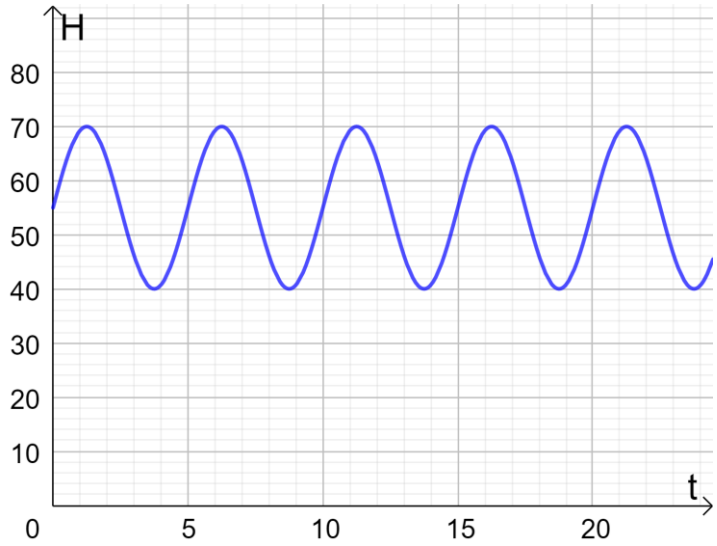
4pt c Compute algebraically the value of Q for which P is maximal.

Question 3 – A windmill

Take a new answer sheet for every question!

Farmer Bert has a windmill on his land. A sensor is mounted on one of the blades of this windmill, which measures the wind speed, among other things.

On a favorable day, the wind blows constantly and the blades of this mill rotate at a constant speed. The figure below shows a graph depicting the relationship between H, the height of the sensor above the ground in meters, and t, the time in seconds.



A formula of the form $H = A + B\sin(Ct)$ fits this figure.

4pt a Use the figure to determine *A*, *B* and *C*.

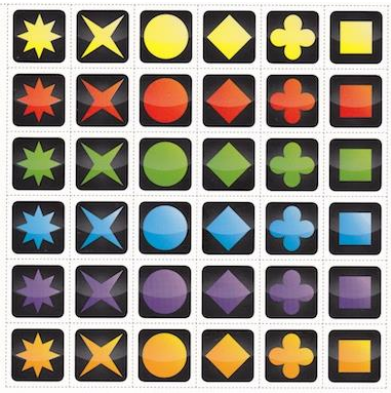
At $t = 0.58$ seconds, the sensor is at a height of 64.99 meters above the ground.

4pt b Compute algebraically the next three times at which the sensor is at a height of 64.99 meters above the ground.

Question 4 – Qwirkle

Take a new answer sheet for every question!

Qwirkle is a tile-based game for 2 to 4 players.
Qwirkle is played with wooden tiles. Each tile is painted with one of 6 shapes (clover, four-point star, eight-point star, square, circle and diamond) in one of 6 colours (yellow, red, green, blue, purple and orange).
The game is played with 3 sets of the 36 tiles depicted in the figure on the right, so in total there are 108 tiles.
At the start of the game, all 108 tiles are in a bag and each player randomly takes 6 tiles from this bag.
(Source: Wikipedia)



Sietse and Hielke are playing this game several times. At the start of the first game, three of the tiles that Sietse takes from the bag are identical (same shape and colour), the other three are all unique (different shapes or colours).

3pt a Compute the number of different orders in which Sietse can place these tiles in a row.

At the start of the second game, the first tile that Hielke takes from the bag is green.

3pt b Compute the probability that the second tile that Hielke takes is green as well.

The tiles are produced by machine and the weight of the tiles is normally distributed with mean $\mu = 5.8$ g and standard deviation $\sigma = 0.3$ g. Regarding the weight, the 108 tiles in one set can be considered to be a random sample from this production.

5pt c Compute the number of tiles in a set of 108 that, according to the thumb rules for the normal distribution, will have a weight between 5.2 g and 6.1 g.

5pt d Compute the probability that 4 of the 6 tiles that Sietse takes from the bag at the start of a game have a weight of more than 5.8 g.

Question 5 – Qwirkle tiles

Take a new answer sheet for every question!

At a quality control of the machine that produces the Qwirkle tiles, the controller tests whether the mean weight of the tiles still is 5.8 g. To test this, the weight of 25 tiles that are produced by the machine is measured. In this testing procedure, it is assumed that the standard deviation of the weight of the tiles still is 0.3 g and the significance level is $\alpha = 0.05$.

2pt a What are the null hypotheses and the alternative hypotheses for this testing procedure?

The mean weight of the 25 tiles is 5.9 g. This results in a p-value of 0.048.

3pt b Determine the parameters μ and σ of the test statistic that is used to compute this p-value.

2pt c What is the conclusion of this testing procedure?
Give a motivation for you answer!

Question 6 is on the next page!

Question 6 – The spread of the coronavirus

Take a new answer sheet for every question!

At the start of the second corona wave, the number of new cases in the Netherlands grew exponentially. In the week from 30 September to 6 October there were 27 485 new cases, in the next week (7 – 13 October) there were 43 903 new cases.

5pt a Compute algebraically the time in days in which the number of new cases would have doubled if the number of new cases had continued to increase exponentially in the following weeks. Give you answer rounded to 2 digits behand the decimal point.

On 14 October, a partial lockdown took effect in the Netherlands. Scientists have developed several models to predict the effect of this partial lockdown. In one of these models, the number of new cases per day is given by the formula

$$N = \frac{30\,000}{1 + 2\mathrm{e}^{0{,}1t}}$$

(t in days, $t = 0$ on 1 November 2020).

5pt b Compute algebraically the date at which according to this model there will be 100 new cases in the Netherlands.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^g\log a + {}^g\log b = {}^g\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a - {}^g\log b = {}^g\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a^p = p \cdot {}^g\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^g\log a = \frac{{}^p\log a}{{}^p\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ with $k = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$ $g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$ $g_r = \mu_T + 2.58\sigma_T$

