Elaborations Example Exam 2 Wiskunde A 2018

Question 1a - 4 points

$9x^3 + 3x^2 = 2x \Leftrightarrow 9x^3 + 3x^2 - 2x = 0 \Leftrightarrow x(9x^2 + 3x - 2) = 0 \Leftrightarrow x = 0 \vee 9x^2 + 3x - 2 = 0$

$9x^2 + 3x - 2 = 0 \Leftrightarrow (3x + 2)(3x - 1) = 0 \Leftrightarrow x = -\frac{2}{3} \vee x = \frac{1}{3}$

Of course, you may also use the quadratic formula: $x = \frac{-3 \pm \sqrt{81}}{18} \Leftrightarrow x = -\frac{2}{3} \vee x = \frac{1}{3}$

Solutions: $x = 0,\ x = -\frac{2}{3},\ x = \frac{1}{3}$

Question 1b - 4 points

$3 \cdot 5^x + 14 = 10 \cdot 5^x \Leftrightarrow 7 \cdot 5^x = 14 \Leftrightarrow 5^x = 2 \Leftrightarrow x = {}^5\log(2) \approx 0.431$

Question 1c - 4 points

$$f'(x) = \frac{2x \cdot (x + 2) - x^2 \cdot 1}{(x + 2)^2} = \frac{x^2 + 4x}{(x + 2)^2}$$

$$\text{Slope} = f'(1) = \frac{1 + 4}{3^2} = \frac{5}{9}$$

Question 1d - 5 points

$$f(x) = y \Leftrightarrow \frac{x^2}{x + 2} = \frac{3}{4}x - \frac{1}{2} \Leftrightarrow x^2 = (x + 2)\left(\frac{3}{4}x - \frac{1}{2}\right) \Leftrightarrow x^2 = \frac{3}{4}x^2 + x - 1$$

$$\Leftrightarrow \frac{1}{4}x^2 - x + 1 = 0 \Leftrightarrow x^2 - 4x + 4 = 0 \Leftrightarrow x = 2$$

Intersection (2,1)

Question 2a - 4 points

${}^{\mathrm{d}R}\!/_{\mathrm{d}Q} = 1 \cdot \left(20 - \frac{1}{5}Q\right) + Q \cdot \left(-\frac{1}{5}\right)$ of $R = 20Q - \frac{1}{5}Q^2$

Both yield ${}^{\mathrm{d}R}\!/_{\mathrm{d}Q} = 20 - \frac{2}{5}Q$

${}^{\mathrm{d}R}\!/_{\mathrm{d}Q} = 0 \Leftrightarrow 20 - \frac{2}{5}Q = 0 \Leftrightarrow \frac{2}{5}Q = 20$

This yields $Q = 20 \cdot \frac{5}{2} = 50$

Question 2b - 4 points

First, substitute $P = 12$ into $P = 20 - \frac{1}{5}Q$

This yields $12 = 20 - \frac{1}{5}Q \Leftrightarrow \frac{1}{5}Q = 8 \Leftrightarrow Q = 40$

Substitution of $Q = 40$ into $Q = \sqrt{5I - 10}$ then yields $5I - 10 = 40^2$

This yields $5I = 1600 - 10 = 1590 \Leftrightarrow I = 318$

Alternative:

Substitution of $Q = \sqrt{5I - 10}$ into $P = 20 - \frac{1}{5}Q$ yields $P = 20 - \frac{1}{5}\sqrt{5I - 10}$

$P = 12$ then yields $12 = 20 - \frac{1}{5}\sqrt{5I - 10} \Leftrightarrow \sqrt{5I - 10} = 40$

This results in $5I - 10 = 40^2 \Leftrightarrow 5I = 1600 - 10 = 1590 \Leftrightarrow I = 318$

Question 2c - 4 points

$W = R - I = Q\left(20 - \frac{1}{5}Q\right) - I$

... $= \sqrt{5I - 10}\left(20 - \sqrt{5I - 10}\right) - I$ *of* $20Q - \frac{1}{5}Q^2 - I$

... $= 20\sqrt{5I - 10} - \frac{1}{5}(5I - 10) - I$

... $= 20\sqrt{5I - 10} - I + 2 - I = 2 - 2I + 20\sqrt{5I - 10}$

Question 2d - 6 points

${}^{\mathrm{d}W}\!/_{\mathrm{d}I} = -2 + 20 \cdot \dfrac{1}{2\sqrt{5I - 10}} \cdot 5 = -2 + \dfrac{50}{\sqrt{5I - 10}}$

${}^{\mathrm{d}W}\!/_{\mathrm{d}I} = 0 \Leftrightarrow \dfrac{50}{\sqrt{5I - 10}} = 2 \Leftrightarrow \sqrt{5I - 10} = \dfrac{50}{2} \Leftrightarrow 5I - 10 = 25^2 \Leftrightarrow 5I = 635 \Leftrightarrow I = 127$

This yields $W_{max} = 2 - 254 + 20\sqrt{625} = -252 + 500 = 248$ euro

Question 3a - 4 points

$\mu_T = \mu_G + \mu_F = 1130.5 + 90.5 = 1221.0$ g

$\sigma_T^2 = \sigma_G^2 + \sigma_F^2 = 25^2 + 3^2 = 634$, so $\sigma_T = \sqrt{634} \approx 25.18$ g

Question 3b - 2 points

$H_0\colon\ \mu = 1040;\ \ H_1\colon\ \mu \neq 1040$

Question 3c - 3 points

The test statistic *T* is normally distributed with $\mu_T = 1040$ and $\sigma_T = \frac{23}{\sqrt{100}} = 2.3$

Question 3d - 3 points

This is a two sided test, so the p-value has to be compared with $\frac{1}{2}\alpha = 0.025$.

$0.041 > 0.025$, so the null hypothesis should not be rejected.

There is not sufficient evidence to support the statement that the volume of the tonic in the bottles is not equal to 1040 ml.

Question 4a - 5 points

Read from the graph: maximum 4.8; minimum 0.4; equilibrium 2.6

Two of these have to be read from the graph.

$A = equilibrium = 2{,}6$

$B = amplitude = 2{,}2$

The period is 12.5 hours, so $C = \frac{2\pi}{12.5} = \frac{4}{25}\pi = 0.16\pi \approx 0.503$

Question 4b - 4 points

The period is 12.5 hours, so at 12.30 hours, the dept of the harbour is also 1.54 m.
Midnight in the night from 1 to 2 August is 1 hour before the graph intersects the equilibrium increasingly. The other times at which the dept is also 1.54 m are therefore 1 hour after the times at which the graph passes the equilibrium decreasingly. The time between the intersections with the equilibrium is a half period, so:

First time: 1 hour ($t = 25$) + $6\frac{1}{4}$ hours (half period) + 1 hour = 8.15 hours

Final time: 12.5 hours after 8.15 = 20.45 hours

The answer can also be found by solving the equation $D(t) = 1.54$ *and checking which of the solutions fulfil the description. However, this method is not a part of the wiskunde A program.*

Question 5a - 4 points

With linear growth, the increase per unit of time is constant.

The increase over the first two days is 45, the increase over the next two days is 720.

This means that there in no linear growth.

Question 5b - 4 points

The growth factor over the first two days is $\frac{48}{3} = 16$, the growth factor over the next two days is also 16, so an exponential growth model fits.

The growth factor over one day is $16^{\frac{1}{2}} = 4$

The number of carriers on day 3 is therefore given by $3 \cdot 4^3 = 48 \cdot 4 = 192$

Question 5c - 4 points

$N'(t) = 5000 \cdot (-4\mathrm{e}^{-0.085t}) \cdot (-0.085) = 170 \cdot \mathrm{e}^{-0.085t}$

$N'(4) = 170 \cdot \mathrm{e}^{-0.085 \cdot 4} = 121$ carriers per day

Question 5d - 5 points

$N(t) = 10\,000 \Leftrightarrow 5000 \cdot (3 - 4 \cdot \mathrm{e}^{-0.085t}) = 10\,000 \Leftrightarrow 3 - 4 \cdot \mathrm{e}^{-0.085t} = 2 \Leftrightarrow \mathrm{e}^{-0.085t} = 0{,}25$

Dit geeft $-0.085t = \ln(0.25) \Leftrightarrow t = \frac{\ln(0.25)}{-0.085}$ =16,31 dagen = 16 dagen en 7 uur.

Question 5e - 3 points

In the long run, the term $\mathrm{e}^{-0.085t}$ becomes practically 0.

The number of carriers therefore becomes 15 000.

This means that there will be 5000 non carriers in the long run.

Question 6a - 3 points

X, the number of tenants that show up $n = 20$ and $p = 0,9$

$P(X = 18) = \binom{20}{18} \cdot 0,9^{18} \cdot 0,1^2 = 0,2852$

Question 6b - 3 points

$P(X = 19) = \binom{20}{19} \cdot 0,9^{19} \cdot 0,1 = 0,27017$

$P(X = 20) = 0,9^{20} = 0,12158$

Question 6c - 5 points

The expected nett revenue if he rents out 20 bungalows is

$40\,000 - 4000 \cdot 0.2702 - 8000 \cdot 0.1216 = 37\,946.40$ euro

The revenue for renting out 18 bungalows is $18 \cdot 2000 = 36\,000$ euro

The expected revenue is higher if he rents out 20 bungalows.

Question 6d - 3 points

$19 \cdot 2000 - 4000 \cdot P(all\ 19\ show\ up) = 38\,000 - 4000 \cdot 0,9^{19} = 37\,459,66$ euro

Extra Question a - 5 points

$\mu_X = 10 \cdot 5 + 4 \cdot 15 = 110$ minutes

$\sigma_X^2 = 10 \cdot 1^2 + 4 \cdot 3^2$, so $\sigma_X = \sqrt{10 \cdot 1^2 + 4 \cdot 3^2} = \sqrt{46}$ (≈ 6.782)

Extra Question b - 3 points

6 minutes $= \mu + \sigma$, 4 minutes $= \mu - \sigma$

According to the thumb rules, 68% of the candidates need between 4 and 6 minutes for a multiple choice question, so 32% needs less than 4 minutes or more than 6 minutes.

Because of the symmetry, half of these, so 16%, need more than 6 minutes.

Extra Question c - 2 points

H_0: $p = 0,8$; H_1: $p < 0,8$

Extra Question d - 6 points

The test statistic *T* is the number of passes in the sample.

T is binomially distributed with $n = 50$ and $p = 0.8$

This yields $\mu_T = np = 50 \cdot 0.8 = 40$ and $\sigma_T = \sqrt{np(1-p)} = \sqrt{50 \cdot 0.8 \cdot 0.2} = \sqrt{8}$

The limit of the rejection area is therefore $g = \mu_T - 1.645\sigma_T = 40 - 1.645 \cdot \sqrt{8} = 35.3$

The outcome of the sample is larger than this limit, so the null hypothesis is not rejected.

So the exam is not harder than last year's exam.