CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 16 December 2024
Time: 13.30 – 16.30
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c d e	4 6 5	3 3 4 3 3	6 3	4 4 4 4	6 1 5	4 4 3 2
Total	15	16	9	16	12	13
$\text{Grade} = \frac{\text{total points scored}}{9} + 1$ You will pass the exam if your grade is at least 5.5 .						

Question 1 – Algebraic computations

Take a new answer sheet for every question!

*When you are asked to perform a computation **algebraically**, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$*.*
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = x^4 + x^3 - 30x^2$.

4pt a Compute algebraically the *x*-coordinates of the intersection points of the graph of f and the *x*-axis.

The function g is given by

$$g(x) = \frac{2x + 3}{x^2 + 2x + 3}$$

6pt b Compute algebraically the coordinates on the graph of g where the tangent line to this graph is horizontal.

The function h is given by $h(x) = 5 \cdot 4^{x+2} + 25$.

5pt c Compute algebraically the value of x for which $h(x) = 225$.
Give the answer rounded to two digits behind the decimal point.

Question 2 – Summer dresses

Take a new answer sheet for every question!

A fashion designer regularly presents new designs for summer dresses, that she thereafter produces on demand. The number of orders Q for a design exclusively depends on the price in Euros P.
The relationship between P and Q is given by the formula $Q = \sqrt{90\,000 - P^2}$.

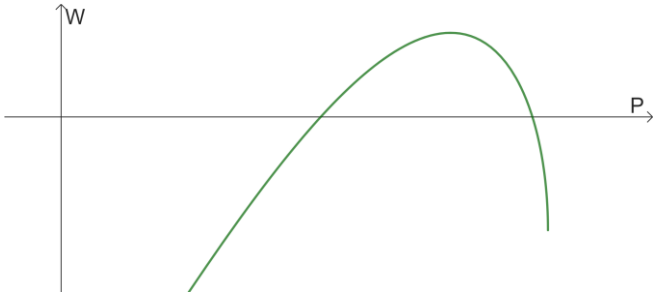
The costs in Euros C for designing and producing the dresses depend on the number of dresses that are produced, which is, like indicated above, equal to Q, the number of dresses that are ordered.

The relationship between C and Q is given by the formula $C = 35 \cdot (400 + 3Q)$.

The profit in Euros W that the designer gets from a design, that is the revenue of the sale of the dresses $(= PQ)$ minus the costs for designing and producing, is given by

$$W = (P - 105) \cdot \sqrt{90\,000 - P^2} - 14\,000$$

In the figure below, a part of the graph that represents the relationship between W and P is shown.



3pt a Show how the formula for W follows from the formulas for Q and C.

3pt b Compute algebraically the profit if 84 dresses are sold.

The derivative of the profit function can be written as

$$\frac{\mathrm{d}W}{\mathrm{d}P} = \frac{90\,000 - 2P^2 + 105P}{\sqrt{90\,000 - P^2}}$$

4pt c Show that this is true.

3pt d Compute algebraically the price at which the profit from a design is maximal.

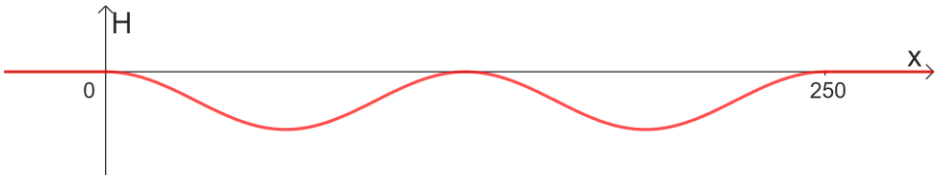
3pt e What is the maximal possible price that the designer can ask for a dress according to the formulas given above?

Question 3 – A reverse speed bump

Take a new answer sheet for every question!

In some cycle paths, to limit the speed of cyclists, there is a double hollow, which works like a reverse speed bump. Here you can see such a hollow on the Statenpad in Ede.

This double hollow has the shape of a sinus function. The distance between the beginning and the end of the double hollow is 250 cm.
The deepest points of the double hollow are 20 cm below the original level of the cycle path. See the figure below.



A formula of the form $H = a + b\sin(c(x - d))$ fits this figure,
with H and x in cm.

6pt a Compute algebraically the values of a, b, c and d in this formula.

At $x = 15.8$, the cycle path is 3.0 cm below its original level.

3pt b Compute algebraically the other three values of x for which the cycle path is 3.0 cm below its original value.

Question 4 – Chocolate bars

Take a new answer sheet for every question!

A supermarket sells mini chocolate bars in bags. Each bag contains 4 bars of milk chocolate, 3 bars of dark chocolate and 2 bars of white chocolate.

Eric buys one of these bags and he eats three randomly chosen bars from this bag.

4pt a Compute the probability that Eric eats exactly one bar of each type.

4pt b Compute the probability that Eric eats at least one bar of white chocolate.

Martin buys 10 of these bags and he eats one randomly chosen bar from each of these bags.

4pt c Compute the probability that Martin eats exactly 5 bars of milk chocolate.

4pt d Compute the expectation and the standard deviation of the number of bars of white chocolate that Martin eats.

More about these chocolate bars in question 5 on the next page.

Question 5 – The weight of the chocolate bars

Take a new answer sheet for every question!

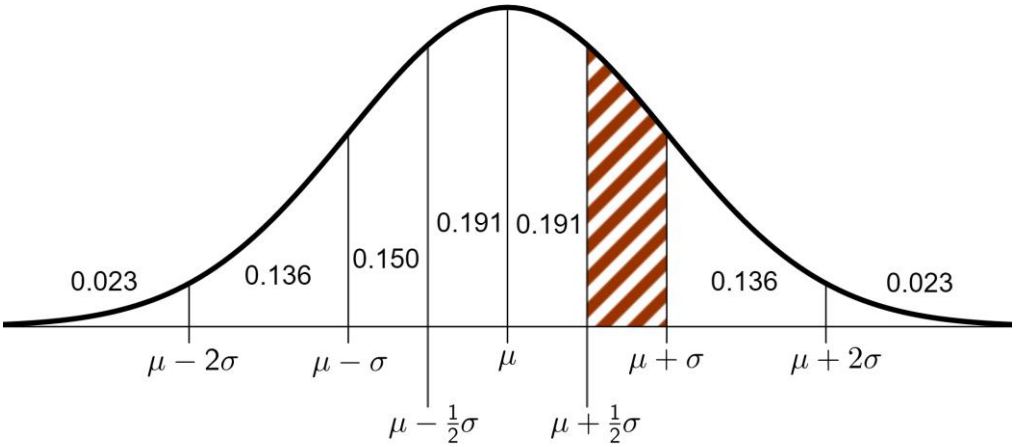
According to the supermarket, the weight of all types of chocolate bars from question 4 is normally distributed with a mean of $\mu = 21.0$ g and a standard deviation of $\sigma = 0.5$ g.

6pt a Use the figure at the bottom of this page to calculate, based on the supermarket's claim, what percentage of the bags containing 9 chocolate bars have a total weight of more than 190.5 g.

A consumer organization suspects that the average weight of the chocolate bars is less than 21 g. To test this, the consumer organization weighs 9 randomly selected chocolate bars. In this test procedure, a significance level of $\alpha = 0.05$ is taken and the standard deviation of the weight of one chocolate bar is assumed to be equal to $\sigma = 0.5$ g.

1pt b State the null hypothesis and the alternative hypothesis for this test procedure.

5pt c What is the conclusion of this test procedure if the average weight of these 9 chocolate bars is 20.5 g ?



A normal probability distribution X
The area of the shaded region represents $P\left(\mu + \frac{1}{2}\sigma < X < \mu + \sigma\right) = 0.150$

Question 6 – The salt concentration in the IJsselmeer

Take a new answer sheet for every question!

The IJsselmeer was created in 1932 by closing off the Zuiderzee with the Afsluitdijk. From that moment on, the salt concentration in the IJsselmeer gradually decreased. This salt concentration is well approximated by the formula

$$C = \frac{58\mathrm{e}^{-t} + 1}{10}$$

In this formula, C is the salt concentration in kg/m^3 and t is the time in years, with $t = 0$ in 1932.

4pt a Compute algebraically the percentage by which the salt concentration in the IJsselmeer decreased in the first year after the closure of the Zuiderzee.

4pt b Compute algebraically the time at which the salt concentration in the IJsselmeer was equal to 3 kg/m^3.

3pt c Compute algebraically the speed in kg/m^3 per year at which the salt concentration decreased at $t = 2$.

After a number of years, the salt concentration in the IJsselmeer has become equal to the salt concentration in the IJssel, the main river that flows into the IJsselmeer.

2pt d Compute algebraically the salt concentration in the IJssel, which is the salt concentration in the IJsselmeer after a number of years.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\big(g(x)\big)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\big(g(x)\big) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^g\log a + {}^g\log b = {}^g\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a - {}^g\log b = {}^g\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a^p = p \cdot {}^g\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^g\log a = \frac{{}^p\log a}{{}^p\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ with $k = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$ $g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$ $g_r = \mu_T + 2.58\sigma_T$

