CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 22 July 2024
Time: 13.30 – 16.30
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c d	5 4 4 4	4 5 5 2	4 5	5 2 4 3	4 5 4	4 6 2
Total	17	16	9	14	13	12

$$\text{Grade} = \frac{\text{total points scored}}{9} + 1$$

You will pass the exam if your grade is at least 5.5 .

Question 1 – Algebraic computations

Take a new answer sheet for every question!

When you are asked to perform a computation ***algebraically****, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$.
Unless stated otherwise, all computations in this exam have to be performed algebraically.

Given are the functions $f(x) = x^3 - 7x^2 + 15x + 4$ and $g(x) = 3x + 4$.

5pt a Compute algebraically the coordinates of the intersections of the graphs of f and g.

The function h is given by $h(x) = \sqrt{x^2 + 5}$.

4pt b Compute algebraically the slope of the graph of h in point $(2, 3)$.

P is the intersection of the graph of h and the line with equation $x + y = 3$.

4pt c Compute algebraically the coordinates of point *P*.

The variables G and t have a relationship, for which the table below holds:

t	G
3	1.2
7	3.6
10	6.0

4pt d Investigate whether this relationship can be linear, exponential, or neither of these.

Question 2 – Spread of a viral disease

Take a new answer sheet for every question!

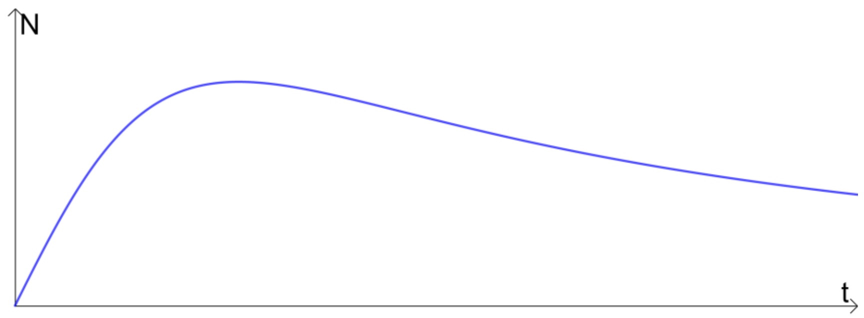
When a new virus emerges, the number of infections often increases rapidly at first, then gradually decreases again. For example, in the 1980s the world was confronted with the outbreak of the HIV virus.

The health centre of country C drew up models that show the relationship between the number of new infections per year in country C (N) and the time t in years with $t = 0$ on 1 January 1980.

In a first model, this relationship is given by the formula

$$N = \frac{50\,000\,t}{t^2 + 25}$$

The figure below shows a graph that represents this relationship.



4pt a Compute algebraically in which years the number of new infections per year is equal to 4000 according to this model.

5pt b Use the derivative $\frac{\mathrm{d}N}{\mathrm{d}t}$ to compute algebraically the maximal number of new infections per year in country C according to this first model.

In a second model, the relationship between N and t is given by the formula

$$N = t \cdot \mathrm{e}^{8-0.16t}$$

In this second model, the slope of the corresponding graph at $t = 5$ is equal to $0.2\mathrm{e}^{7.2}$.

5pt c Show algebraically that this is true.

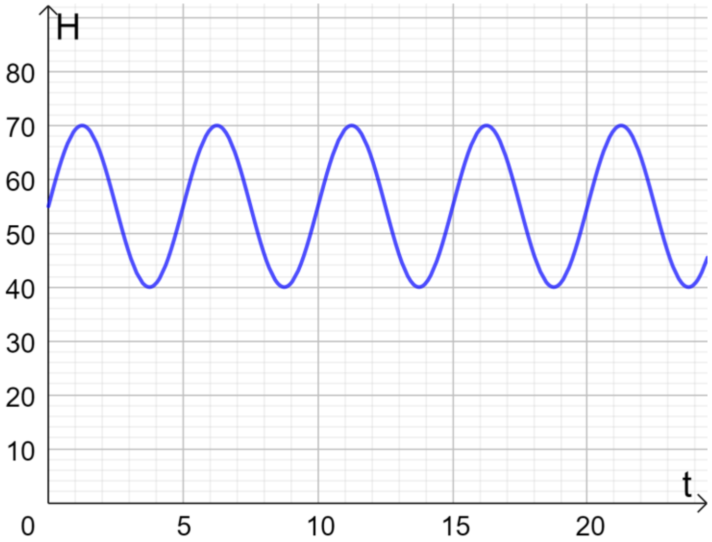
2pt d Is the peak of the number of new infections according to the second model earlier or later than $t = 5$?
Explain your answer.

Question 3 – A windmill

Take a new answer sheet for every question!

Farmer Fred has a windmill on his land. A sensor is mounted on one of the blades of this windmill, which measures the wind speed, among other things.

On a favorable day, the wind blows constantly and the blades of this mill rotate at a constant speed. The figure below shows the graph that represents the relationship between H, the height of the sensor above the ground in meters, and t, the time in seconds.



A formula of the form $H = a + b\sin(ct)$ fits this figure.

4pt a Use the figure to determine values for a, b and c.

For another windmill, the relationship between the height of the sensor (in meters above the ground) and the time (in seconds) is given by the formula

$$H = 55 - 18\sin\big(0.25\pi(t - 1.5)\big)$$

5pt b Compute algebraically the first three times after $t = 0$ at which the sensor is at its highest point.

Question 4 – European elections

Take a new answer sheet for every question!

In the recent European elections in country C, 30% of the electorate voted for a pro-European party, 20% of the electorate voted for an Eurosceptic party and 50% of the electorate did not vote at all. A talk show invites three randomly selected members of the electorate to discuss the results of these elections.

5pt a Compute the probability that they invite one member of the electorate from each category (pro-European, Eurosceptic or non-voter).

The leader of one of the Eurosceptic parties claims, that more than 60% of the non-voters would have voted for an Eurosceptic party if they had been forced to cast a vote. To test this, the talk show asks 100 randomly selected non-voters what they would have voted if they had been forced to cast a vote. 64 of these non-voters say that they would have voted for a Eurosceptic party. In this test procedure, they take a significance level of $\alpha = 0.05$.

2pt b State the null hypothesis and the alternative hypothesis for this test procedure.

The probability that 64 out of 100 randomly selected non-voters would have voted for an Eurosceptic party if 60% of all non-voters would have voted for an Eurosceptic party is approximately equal to 0,06.

4pt c Compute this probability rounded to 6 digits behind the decimal point.

3pt d Can you draw a conclusion for this testing procedure?
If so, motivate this conclusion. If not, explain why.

Question 5 – Conference pears

Take a new answer sheet for every question!

The conference pear is a variety of pear that was introduced at the National British Pear Conference in London in 1885, where it won first prize.
The weight of conference pears is normally distributed with an average of $\mu = 226$ g and a standard deviation of $\sigma = 4$ g.

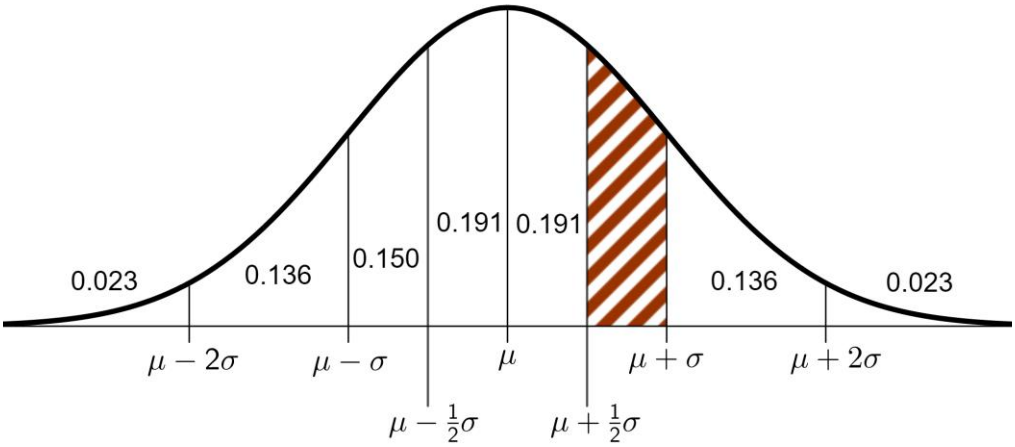
4pt a Use the figure on the bottom of this page to compute the percentage of conference pears that has a weight between 218 g and 230 g.

A super market sells conference pears in boxes that contain 10 pears.
The weight of the empty boxes is normally distributed with an average of $\mu = 53$ g and a standard deviation of $\sigma = 3$ g.

5pt b Use the figure on the bottom of this page to compute the percentage of these boxes that has a total weight (box + 10 pears) of less than 2300 g.

Eva buys one of these boxes. Unfortunately, 4 of the 10 pears in this box are rotten.
She randomly picks two pears from this box.

4pt c Compute the probability that at least one of these two pears is rotten.



A normal probability distribution X
The area of the shaded region represents $P\left(\mu + \frac{1}{2}\sigma < X < \mu + \sigma\right) = 0.150$

Question 6 – A new payment card for public transport

Take a new answer sheet for every question!

In country C, a new payment card for public transport was introduced on 1 January 2010. The number of users of this card is given by the formula

$$N = \frac{30}{2 + 3\mathrm{e}^{-0.2t}}$$

In this formula, N is the number of users of this card in millions and t is the time in years, with $t = 0$ on 1 January 2010.

4pt a Compute algebraically the percentage by which the number of users of this card increased between 1 January 2010 and 1 January 2011.

6pt b Compute algebraically the time (year and month) at which this card had 10 million users.

2pt c Compute the maximal number of users of this card according to the formula given above.

End of the exam.

When you have finished the exam, check whether your ***name*** *and the* ***question number*** *are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

What should ***not*** *be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^{g}\log a + {}^{g}\log b = {}^{g}\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a - {}^{g}\log b = {}^{g}\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a^p = p \cdot {}^{g}\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^{g}\log a = \frac{{}^{p}\log a}{{}^{p}\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If X and Y are any random variables, then: $E(X+Y)=E(X)+E(Y)$
If furthermore X and Y are independent, then: $\sigma(X+Y)=\sqrt{\sigma^2(X)+\sigma^2(Y)}$

$\sqrt{n}$-law:
For n independent repetitions of the same experiment where the result of each experiment is a random variable X, the sum of the results is a random variable S and the mean of the results is a random variable $\overline{X}$.

$E(S)=n\cdot E(X)$ $\qquad$ $\sigma(S)=\sqrt{n}\cdot\sigma(X)$

$E(\overline{X})=E(X)$ $\qquad$ $\sigma(\overline{X})=\dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If X has a binomial distribution with parameters n (number of experiments) and p (probability of success at each experiment), then

$$P(X=k)=\binom{n}{k}\cdot p^k\cdot(1-p)^{n-k} \quad \text{with} \quad k=0,1,2,\dots,n$$

Expected value: $E(X)=np$ $\qquad$ Standard deviation: $\sigma(X)=\sqrt{n\cdot p\cdot(1-p)}$

n and p are the parameters of the binomial distribution

Normal Distribution

If X is a normally distributed random variable with mean μ and standard deviation σ, then

$Z=\frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X<g)=P\left(Z<\frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic T is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g=\mu_T-1.645\sigma_T$	$g=\mu_T+1.645\sigma_T$	$g_l=\mu_T-1.96\sigma_T$
			$g_r=\mu_T+1.96\sigma_T$
0.01	$g=\mu_T-2.33\sigma_T$	$g=\mu_T+2.33\sigma_T$	$g_l=\mu_T-2.58\sigma_T$
			$g_r=\mu_T+2.58\sigma_T$

