CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 19 April 2024
Time: 13.30 – 16.30
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c e	6 5 6	6 5 6	3 4 3 5	4 3 2 5	4 6	3 5
Total	17	17	15	14	10	8
$\text{Grade} = \frac{\text{total points scored}}{9} + 1$ You will pass the exam if your grade is at least 5.5 .						

Question 1 – Algebraic computations

Take a new answer sheet for every question!

When you are asked to perform a computation ***algebraically****, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$.
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = (x^2 - 2)(x^2 - 4)$.
The function g is given by $g(x) = x^3 + 8$.

6pt a Compute algebraically the coordinates of the intersection(s) of the graphs of the functions f and g.

The function h is given by

$$h(x) = \sqrt[3]{x} - \frac{16}{x}$$

The point $H(8,0)$ is on the graph of h.
You do not have to show this.

5pt b Compute algebraically the slope of the tangent line to the graph of h in point H.

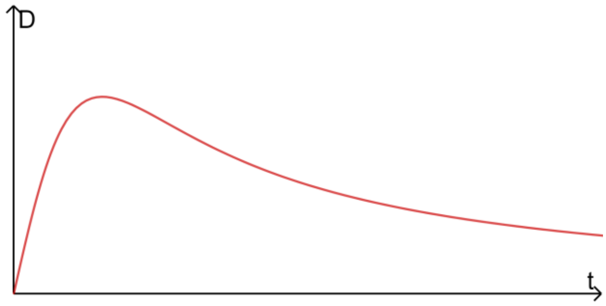
The function k is given by $k(x) = \sqrt{3x^2 + 24}$.

6pt c Compute algebraically the value(s) of a for which the tangent line to the graph of k in the point $A\left(a, k(a)\right)$ is parallel to the line $y = -x$.

Question 2 – Pizza!

Take a new answer sheet for every question!

A large pizza company launches a new pizza: The San Francisco Special.
At first, the demand for the San Francisco Special rises sharply, but after a while, the demand drops. This is shown in the figure below.



The relationship between D, the demand in millions per day (worldwide!), and t, the time in weeks with $t = 0$ at the launch of the San Francisco Special, can be approximated by the formula

$$D = \frac{165t}{2t^2 + 450}$$

6pt **a** Use the derivative $\frac{dD}{dt}$ to compute algebraically the maximal demand per day for the San Francisco Special according to this formula.

When the demand per day drops below 1 million, the San Francisco Special will be discontinued and another pizza will be launched: The Los Angeles Delight.

5pt **b** Compute algebraically how much time after its launch the San Francisco Special is discontinued according to the formula given above.
Give your answer rounded to whole days.

The expected demand for the Los Angeles Delight is given by the formula

$$E = 0.4t \cdot \mathrm{e}^{-0.05t}$$

In this formula, E is the expected demand for the Los Angeles Delight in millions per day and t is the time in weeks with $t = 0$ at the launch of this pizza.

6pt **c** Use the derivative $\frac{dE}{dt}$ to compute algebraically the maximal expected demand per day for the Los Angeles Delight according to this formula.

Question 3 – A special game with a regular die.

Take a new answer sheet for every question!

Joe and Donald play a special game with a regular die.

At each turn, both players throw the die. If the outcome of his throw is even, the player gets this outcome as his points. However, if the outcome of the throw is odd, the player gets twice the outcome as his points.

The number of points that one player gets in one turn, is a random variable X.
The probabilities for X are in the table below.

Points x	2	4	6	10
Probability $P(X = x)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

3pt a Compute $E(X)$, the expected number of points that a player gets in each turn.

4pt b Compute the probability that the sum of the points that Joe and Donald get in a turn, is exactly 8.

The probability that Joe and Donald get the same number of points in a turn is $\frac{5}{18}$.

3pt c Show that this is true.

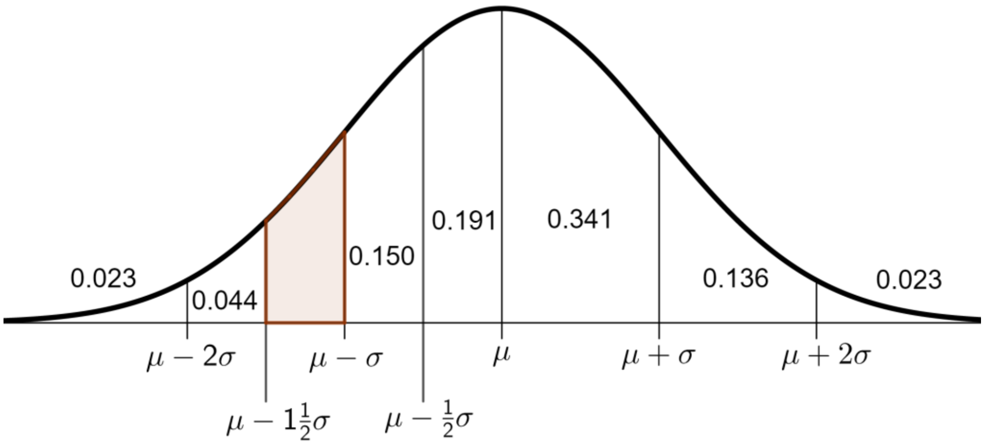
5pt d Compute the probability that Joe gets more points than Donald in exactly four of the first ten turns.

Question 4 – Peanut butter

Take a new answer sheet for every question!

The Vecal factory sells peanut butter in jars.
The weight of the peanut butter in these jars is normally distributed with an average of $\mu = 450$ g and a standard deviation of $\sigma = 4$ g.

4pt a Use the figure below to compute the percentage of these jars that contain between 442 and 452 g peanut butter.



A normal probability distribution X
The area of the shaded region represents $P\left(\mu - 1\frac{1}{2}\sigma < X < \mu - \sigma\right) = 0.092$

The weight of the empty jars is normally distributed with an average of $\mu = 100$ g and a standard deviation of $\sigma = 1$ g.

3pt b Compute the average and the standard deviation of the total weight of a jar filled with peanut butter.

At a quality control, the Vecal factory wants to test whether the average weight of the peanut butter in a jar is indeed 450 g. For this test, they measure the weight of the peanut butter in 100 jars that are filled independently of each other. They assume that the weight is normally distributed with a standard deviation of $\sigma = 4$ g and they take a significance level of $\alpha = 0.05$.

2pt c State the null hypothesis and the alternative hypothesis for this test procedure.

5pt d What is the conclusion of this test procedure if the average weight of the peanut butter in the 100 jars from the test is 450.7 g?

Question 5 – Two wave pools

Take a new answer sheet for every question!

A wave pool is a swimming pool in which waves are generated by valves. If these valves are in motion, the depth of the water at a certain point is a periodic function. For a certain wave pool, this function is given by

$$D = 3 + 0.5\sin\left(\tfrac{1}{18}\pi \cdot (4t + 3)\right)$$

In this formula, *D* is the depth in meters of the water (which is the height of the water in relation to the bottom of the pool), and *t* is the time in seconds.

4pt **a** Compute algebraically the first three times after $t = 0$ at which the depth of the water is 3.5 meter according to this formula.

For another wave pool, the depth in meters of the water at a certain point as a function of the time in seconds, is shown in the graph below.

This graph represents a function with a formula of the form

$$D = a + b \cdot \sin\big(c \cdot (t - d)\big)$$

6pt **b** Compute possible values for a, b, c and d in this formula.

Question 6 – Two growth models

Take a new answer sheet for every question!

The number of users of the social media platform *Y* is given by the formula

$$N_Y = 2.5 \cdot 1.5^t$$

In this formula, N_Y is the number of users in millions and t is the time in years.

3pt a Compute algebraically the time at which platform *Y* will have 100 million users.

The number of users of the social media platform *Z* is given by the formula

$$N_Z = \frac{200}{1 + 4\mathrm{e}^{-0.15t}}$$

In this formula, N_Z is the number of users in millions and t is the time in years.

5pt b Compute algebraically the time at which platform *Z* will have 100 million users.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^{g}\log a + {}^{g}\log b = {}^{g}\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a - {}^{g}\log b = {}^{g}\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a^p = p \cdot {}^{g}\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^{g}\log a = \frac{{}^{p}\log a}{{}^{p}\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If X and Y are any random variables, then: $E(X+Y)=E(X)+E(Y)$
If furthermore X and Y are independent, then: $\sigma(X+Y)=\sqrt{\sigma^2(X)+\sigma^2(Y)}$

$\sqrt{n}$-law:
For n independent repetitions of the same experiment where the result of each experiment is a random variable X, the sum of the results is a random variable S and the mean of the results is a random variable $\overline{X}$.

$E(S)=n\cdot E(X)$ $\qquad$ $\sigma(S)=\sqrt{n}\cdot\sigma(X)$

$E(\overline{X})=E(X)$ $\qquad$ $\sigma(\overline{X})=\dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If X has a binomial distribution with parameters n (number of experiments) and p (probability of success at each experiment), then

$$P(X=k)=\binom{n}{k}\cdot p^k\cdot(1-p)^{n-k} \quad \text{with} \quad k=0,1,2,\ldots,n$$

Expected value: $E(X)=np$ $\qquad$ Standard deviation: $\sigma(X)=\sqrt{n\cdot p\cdot(1-p)}$

n and p are the parameters of the binomial distribution

Normal Distribution

If X is a normally distributed random variable with mean μ and standard deviation σ, then

$Z=\frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X<g)=P\left(Z<\frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic T is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g=\mu_T-1.645\sigma_T$	$g=\mu_T+1.645\sigma_T$	$g_l=\mu_T-1.96\sigma_T$
			$g_r=\mu_T+1.96\sigma_T$
0.01	$g=\mu_T-2.33\sigma_T$	$g=\mu_T+2.33\sigma_T$	$g_l=\mu_T-2.58\sigma_T$
			$g_r=\mu_T+2.58\sigma_T$

