CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date:	24 July 2023
Time:	13.30 – 16.30
Questions:	6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c d	6 6	5 6	5 5 4	4 5 2 5	3 6 4	4 4 5 2
Total	12	11	14	16	13	15

$$\text{Grade} = \frac{\text{total points scored}}{9} + 1$$

You will pass the exam if your grade is at least 5.5 .

Question 1 – Algebraic computations

Take a new answer sheet for every question!

When you are asked to perform a computation ***algebraically****, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$.
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = 15x^6 - 24x^5 + 10x^4$.

6pt a Compute algebraically the minimal value of $f(x)$.

The function g is given by $g(x) = x \cdot \sqrt{2x^2 + 7}$.

6pt b Compute algebraically an equation for the tangent line to the graph of g in the point $A(1, 3)$.

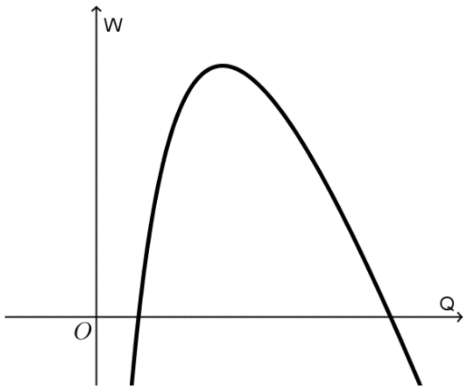
Question 2 – Butcher Baker

Take a new answer sheet for every question!

Butcher Baker sells first quality meat at the Market Square. His brother, a well-known economist, has analyzed the sales figures of Butcher Baker and he has derived the formula below for the relation between the profit Butcher Baker makes in a day and the quantity of meat that he buys in for that day.

$$W = 18 - 3Q - \frac{64}{3Q + 2}$$

In this formula, *W* is the profit in hundreds of euros and *Q* is the quantity of meat bought in for a day in hundreds of kilograms.
In the figure on the right a sketch of the graph of this function is shown.



5pt a Compute algebraically the values of *Q* for which $W = 0$ (the *break-even* values).

6pt b Compute algebraically the maximum profit that Butcher Baker can make in a day.

Question 3 – The Game of the Goose

Take a new answer sheet for every question!

In the Game of the Goose, two or more players move pieces (usually in the form of a goose) around the track pictured below. The number of squares that players move their goose forward is decided by throwing two regular dice.

As you can see, there are a number of special squares, such as square 5, square 6, square 9 and square 12. You can ignore the meaning of these squares in questions a and c.

Bert and Ernie are playing this game. In his first turn, Bert's goose lands on square 4.

5pt a Compute the probability that Ernie's goose lands on a square that is further from the start.

Two of the special squares are the water well (square 31) and the prison (square 52). If a goose lands on one of these squares, the player has to wait until another goose lands on the same square. When there are only two players, there is a 22% probability that one of the players ends in the water well and the other ends in the prison. In that case, the game ends in a draw.

5pt b Assuming that each player has the same probability of winning each game, compute the probability that Bert wins 4 of the first 10 games that they are playing.

Later on, Tommy also joins Bert and Ernie in playing the game.

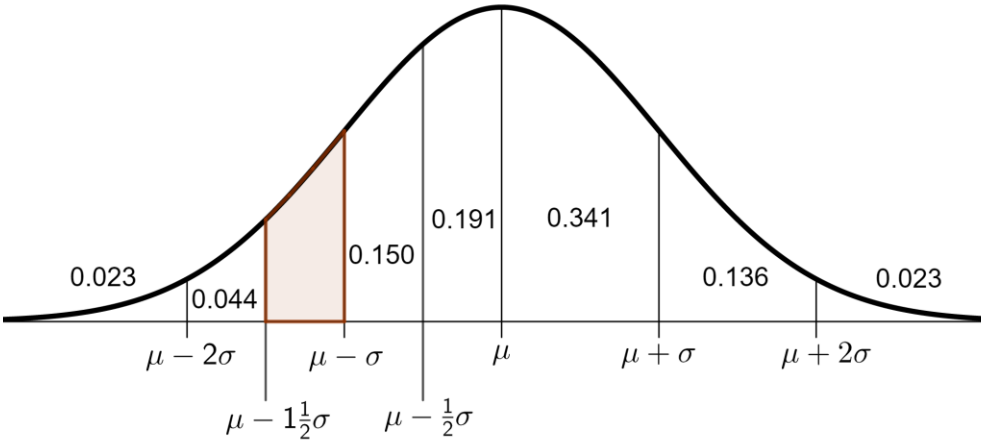
4pt c Compute the probability that their three geese all land on square 7 in the first turn of a game.

Question 4 – Olive Oil

Take a new answer sheet for every question!

The Olivia factory sells olive oil in glass bottles. The volume of the olive oil in these bottles is normally distributed with an average of $\mu = 450$ ml and a standard deviation of $\sigma = 2$ ml.

4pt a Use the figure below to compute the percentage of these bottles that contain between 448 and 451 ml olive oil.



A normal probability distribution X
The area of the shaded region represents $P\left(\mu - 1\frac{1}{2}\sigma < X < \mu - \sigma\right) = 0.092$

At 20℃, the density of this olive oil is 920 kg/m^3. This means that the weight of the olive oil in a bottle from Olivia is normally distributed with an average of $\mu = 414$ g and a standard deviation of $\sigma = 1.84$ g.
The weight of a bottle that is filled with olive oil is normally distributed with an average of $\mu = 500$ g and a standard deviation of $\sigma = 3$ g.

5pt b Compute the average and the standard deviation of the weight of an empty bottle.

At a quality control, the Olivia factory wants to test whether the average volume of the olive oil in a bottle is indeed 450 ml. For this test, they measure the volume of the olive oil in 25 bottles. They assume that the volume is normally distributed with a standard deviation of $\sigma = 2$ ml and they take a significance level of $\alpha = 0.05$.

2pt c State the null hypothesis and the alternative hypothesis for this test procedure.

5pt d What is the conclusion of this test procedure if the average volume of the olive oil in the 25 bottles from the test is 450.7 ml?

Question 5 – UV index and Wolf number

Take a new answer sheet for every question!

The UV index is an international standard measurement of the strength of the sunburn produced by ultraviolet radiation. The higher the UV index, the lower the time one can be exposed to direct sunlight before getting sunburn, see the table below.

UV index (U)	2	5	7
Minutes before getting sunburn (m)	50	20	15

It is obvious that the relationship between the quantities U and m is not linear.

3pt a Investigate whether there is an exponential relationship between U and m.

In the Netherlands, the average value of the UV index is minimal in January and maximal in July. This average can be approximated with a formula of the form

$$U = a + b\sin\left(c(t - d)\right).$$

In this formula, t is the time in months with $t = 0$ on 1 January.

6pt b Find the values of a, b, c and d in this formula, given that the minimal value $U = 0.4$ is reached on 1 January and that the maximal value $U = 7.0$ is reached on 1 July.
Explain your answers.

The Wolf number is a quantity that measures the number of sunspots present on the surface of the Sun. This number can be approximated by the formula

$$W = 87 + 73 \cdot \sin\left(\frac{2\pi}{11}(t - 2.75)\right)$$

In this formula, t is the time in years with $t = 0$ on 1 January 1976.

4pt c Compute algebraically the years between 1976 and 2023 in which the Wolf number was maximal according to this formula.

Question 6 – House warming

Take a new answer sheet for every question!

One of the first things Rob does after returning from a winter holiday is to turn on the heating in his living room. After that moment, the temperature in this room is given by the formula

$$T = 4 \cdot (5 - 3 \cdot \mathrm{e}^{-0{,}5t})$$

In this formula, T is the temperature in ℃ and t is the time in hours, with $t = 0$ at the moment that Rob turns on the heating.

4pt a Compute algebraically by what percentage the value of T rises between $t = 0$ and $t = 2$.

4pt b Use the derivative $\frac{\mathrm{d}T}{\mathrm{d}t}$ to compute algebraically the rate in ℃ per hour at which the value of T rises at $t = 1$.

5pt c Compute algebraically the time at which the temperature is equal to 16℃. Give your answer in minutes accurately.

2pt d Compute the maximal temperature in Rob's living room according to the formula given above.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	function	derivative function
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	conditions
${}^{g}\log a + {}^{g}\log b = {}^{g}\log ab$	$g > 0,\ g \neq 1,\ a > 0,\ b > 0$
${}^{g}\log a - {}^{g}\log b = {}^{g}\log \frac{a}{b}$	$g > 0,\ g \neq 1,\ a > 0,\ b > 0$
${}^{g}\log a^{p} = p \cdot {}^{g}\log a$	$g > 0,\ g \neq 1,\ a > 0$
${}^{g}\log a = \frac{{}^{p}\log a}{{}^{p}\log g}$	$g > 0,\ g \neq 1,\ a > 0,\ p > 0,\ p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If X and Y are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore X and Y are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For n independent repetitions of the same experiment where the result of each experiment is a random variable X, the sum of the results is a random variable S and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad \sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad \sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If X has a binomial distribution with parameters n (number of experiments) and p (probability of success at each experiment), then

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \quad \text{with} \quad k = 0, 1, 2, \ldots, n$$

Expected value: $E(X) = np$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and p are the parameters of the binomial distribution

Normal Distribution

If X is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic T is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$
			$g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$
			$g_r = \mu_T + 2.58\sigma_T$

