CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 20 April 2023
Time: 13.30 – 16.00 (150 minutes)
Questions: 5

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Note that because of the limited availability of the exam hall, the time of this exam is reduced to 2.5 hours (150 minutes). Of course, the number of questions is reduced too.

Points that can be scored for each item:					
Question	1	2	3	4	5
a b c d	6 6 4	5 5 5	4 4 4 4	3 4 6	5 2 5
Total	16	15	16	13	12
$\text{Grade} = \frac{\text{total points scored}}{8} + 1$ You will pass the exam if your grade is at least 5.5 .					

Question 1 – Algebraic computations

Take a new answer sheet for every question!

*When you are asked to perform a computation **algebraically**, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$*.*
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = x^4 - 4x^3 - 8x^2$.

6pt a Compute algebraically the minimal value of $f(x)$.

The function g is given by $g(x) = \sqrt{4x^2 + 48}$.

6pt b Compute algebraically the value(s) of a for which the tangent line to the graph of g in the point $A(a, g(a))$ is parallel to the line $y = -x$.

The relationship between the quantities Q and R is given by the formula

$$\log(R) = 2\log(Q) + 3$$

This formula can be transformed into a formula of the form

$$R = c \cdot Q^d$$

4pt c Compute algebraically the values of c and d in this second formula.

Question 2 – The spread of a disease

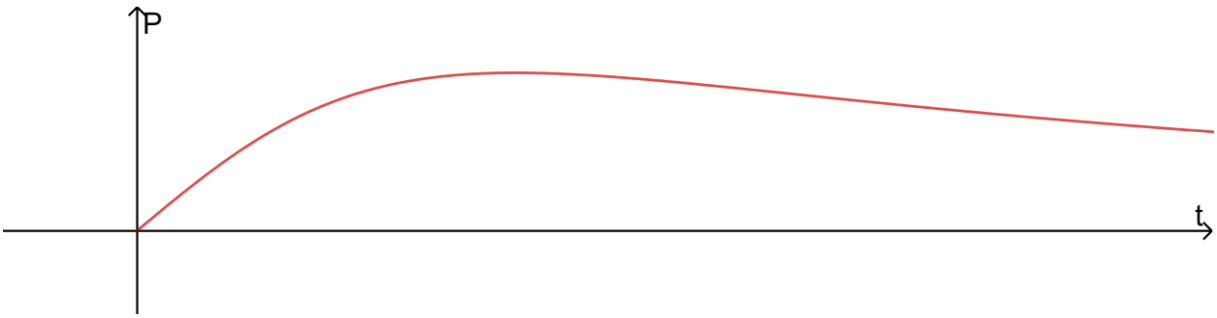
Take a new answer sheet for every question!

When a new viral disease appears, the percentage of the population affected by this disease often rises at first, but later on, it will fall again. For a certain disease, two models are introduced for the spread of this disease.

In the first model, P, the percentage of the population that is affected by the disease, is given by the formula

$$P = \frac{30t}{t^2 + 36}$$

In this formula, t is the time in weeks after the disease was first discovered.
In the figure below, the graph is shown that depicts the relationship between P and t.



5pt a Compute algebraically the number of weeks in which more than 2% of the population is affected by this disease according to this first model.

5pt b Compute algebraically the maximal percentage of the population that was affected by this disease according to this first model.

In the second model, P the percentage of the population that is affected by the disease, is given by the formula

$$P = 1.4t \cdot \mathrm{e}^{-0.2t}$$

In this formula, t is again the time in weeks after the disease was first discovered.

5pt c Compute algebraically the time at which the percentage of the population that is affected by this disease is maximal according to the second model.

Question 3 – Pachisi

Take a new answer sheet for every question!

Pachisi is an ancient Indian board game that can be seen as a precursor of the British game Ludo and the Dutch game *Mens-Erger-Je-Niet*. Instead of a die, the number of moves that a player can do at each turn is decided by a throw of six cowrie shells, as seen in the photo on the right.

In the table below, the possible outcomes of each throw and their probabilities for a certain set of six cowrie shells are shown.

Number of shells with opening on top	6	5	4	3	2	1	0
Number of moves	6	5	4	3	2	10	25
Probability	0.10	0.13	0.17	0.20	0.25	0.10	0.05

Albert plays this game with his wife Betty.

4pt a Compute the expected number of moves that Albert gets from his first throw with the six cowrie shells.

4pt b Compute the probability that Albert gets precisely 12 moves from his first <u>two</u> throws with the six cowrie shells.

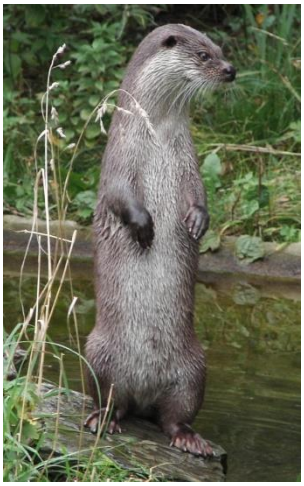
4pt c Compute the probability that Albert gets at least 11 moves from his first <u>five</u> throws with the six cowrie shells.

Later, their daughters, Carla and Daisy, and their sons, Eddie and Freddie, join Albert and Betty in playing this game. They decide to randomly form two teams of three.

4pt d Compute the probability that one of the teams is all male and the other team is all female.

Question 4 – Otter populations

Take a new answer sheet for every question!



The Eurasian otter is a semiaquatic mammal native to Europe and Asia. In The Netherlands, the otter was almost extinct, but in recent years, populations are thriving again.

In one nature reserve, there were 45 otters in 2010, 60 otters in 2015 and 80 otters in 2020.

3pt a What kind of growth model best fits these data, a linear model or an exponential model?
Explain your answer!

In a second nature reserve, the number of otters increases by 4% every year.

4pt b Compute the time in months in which the population of otters in this second nature reserve will double if this population continues to grow by 4% per year.

In a third nature reserve, the number of otters was so large, that there were not enough fish left for them to feed on. Therefore, the number of otters decreased, but then the number of fish increased again, followed by an increase in the number of otters. And then the number of fish decreased again and the process repeated itself. In the years 2010, 2014, 2018 and 2022, the number of otters in this nature reserve reached its maximum of 120. In the years 2012, 2016 and 2020 the number of otters in this nature reserve reached its minimum of 70.

The number of otters in this nature reserve can be modelled by a formula of the form

$$N = a + b \cdot \sin\big(c(t - d)\big)$$

In this formula, N is is the number of otters in this nature reserve and t is the time in years, with $t = 0$ in 2010.

6pt c Find the values of a, b, c and d in this formula.
Explain your answers.

Question 5 – Bonobos

Take a new answer sheet for every question!

The **bonobo** (***Pan paniscus***), also historically called the **pygmy chimpanzee**, is an endangered great ape and one of the two species making up the genus *Pan,* the other being the common chimpanzee (*Pan troglodytes*). It is found in a 500,000 km^2 area of the Congo Basin in the Democratic Republic of the Congo, Central Africa. *Source: Wikipedia.*

The length of adult bonobos in the wild is normally distributed with an average of $\mu = 76.5$ cm and a standard deviation of $\sigma = 2.2$ cm.

5pt a Use the rules of thumb to compute the percentage of these apes that have a length between 72.1 cm and 78.7 cm.

A zoo has a large population of bonobos. They want to test whether the mean length of their bonobos differs from the mean length of the bonobos in the wild. For this test, they measure the length of 16 adult bonobos in the zoo. They assume that the length of the bonobos in the zoo is normally distributed with a standard deviation of $\sigma = 2.2$ cm and they take a significance level of $\alpha = 0.05$.

2pt b State the null hypothesis and the alternative hypothesis for this test procedure.

5pt c What is the conclusion of this test procedure if the average length of the 16 bonobos from the zoo is 75.5 cm?

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\big(g(x)\big)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\big(g(x)\big) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^g\log a + {}^g\log b = {}^g\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a - {}^g\log b = {}^g\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a^p = p \cdot {}^g\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^g\log a = \frac{{}^p\log a}{{}^p\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ with $k = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$ $g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$ $g_r = \mu_T + 2.58\sigma_T$

