CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 22 July 2022
Time: 13.30 – 16.30
Questions: 5

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:					
Question	1	2	3	4	5
a b c d e	6 6 5 5	4 5 4 5	3 4 1 4 3	4 3 4 5	2 4 4
Total	22	18	15	16	10
Grade $= \frac{\text{total points scored}}{9} + 1$ You will pass the exam if your grade is at least 5.5 .					

Question 1 – Algebraic computations

Take a new answer sheet for every question!

*When you are asked to perform a computation **algebraically**, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like $\sqrt{2}$ and $\log(3)$.*
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = x^4 + 7x^3 - 9x^2 + 2x - 7$.
The line ℓ is given by the equation $y = 2x + 4$.

6pt a Compute algebraically the values of a for which the tangent line of the graph of f in the point $(a, f(a))$ is parallel to line ℓ.

The function g is given by

$$g(x) = \frac{x}{(x-3)^2}$$

6pt b Compute algebraically the coordinates of the point(s) on the graph of g in which the tangent line to this graph is horizontal.

Given is a geometric sequence with second term $u_2 = 5120$ and fifth term $u_5 = 640$.

5pt c Compute algebraically the sum of the first ten terms of this sequence, that is $u_1 + u_2 + \cdots + u_9 + u_{10}$.

The relationship between the quantities P and L is given by the formula

$$\log(P) = 0.16\log(L) - 0.20$$

This formula can be transformed into a formula of the form

$$L = a \cdot P^b$$

5pt d Compute algebraically the values of a and b in this second formula.

Question 2 – Growth models

Take a new answer sheet for every question!

A biologist is studying the growth of a culture of bacteria. At a certain point in time $(t = 0)$ the weight of this culture is 576 milligrams. One hour later $(t = 1)$ the weight of this culture is 720 milligrams.

The biologist first models this growth process with a formula of the form

$$W = a \cdot \mathrm{e}^{bt}$$

In this formula, W is the weight of the culture in milligrams and t is the time in hours.

4pt a Compute algebraically the values of a and b that fit the data given above.

Next, the biologist models this growth process with the formula

$$W = (576 - 216t) \cdot \mathrm{e}^{0.69315t}$$

with W and t as indicated above.

5pt b Compute algebraically the growth rate $\frac{\mathrm{d}W}{\mathrm{d}t}$ at $t = 1$.
Give your answer in milligrams per hour accurately.

The biologist examines the influence of the temperature on the growth of bacteria. He finds the following relationship between T, the temperature of a culture of bacteria in degrees Celsius, and P, the growth percentage of the weight of this culture over a period of one hour:

$$P = \sqrt{111T + 54T^2 - T^3} - 100$$

This formula is valid for temperatures between $15°\mathrm{C}$ and $50°\mathrm{C}$.

4pt c Compute in milligrams accurately the weight of a culture of bacteria after one hour, given that the weight at the beginning of this hour is 110 milligrams and that the temperature is kept at $25°\mathrm{C}$ during this hour.

5pt d Compute algebraically the temperature between $15°\mathrm{C}$ and $50°\mathrm{C}$ for which the growth percentage of the weight of a culture is maximal.

Question 3 – Whistle or tails

Take a new answer sheet for every question!

Diederik is a referee in amateur soccer. Before a game he usually uses a coin to determine which team kicks off. One day Diederik forgets to take a coin and he uses his whistle to toss. He lets the teams choose between “hole up”, like in the picture, or not (so hole down or on the side).

Later, Diederik wonders whether the probability of “hole up” is equal to 0.5 or not. He decides to throw the whistle six times and to note the result of each throw with U (up) or N (not). In this way, he gets a series like for example UUNNUN.

3pt a How many different series of six are possible?

If in a series a U is followed by an N or if an N is followed by a U, we call it a change. For example, the series UUUUUU has no changes, but the series UNUUUN has three changes.

4pt b How many series of six have two changes?

Based on his experience to date, Diederik has the idea that the probability of “hole up” is smaller than 0.5. He decides to test this by throwing the whistle 50 times. The whistle ends “hole up” in 20 of this 50 throws.

1pt c State the null hypothesis and the alternative hypothesis for this test procedure.

4pt d Compute the probability to find exactly 20 times “hole up” in 50 throws if the probability of “hole up” is equal to 0.5.

3pt e Can you draw a conclusion from this testing procedure?
If so, what is this conclusion and why?
If not, point out all the information that you furthermore need to be able to draw a conclusion.

Question 4 – Enough time for the exam?

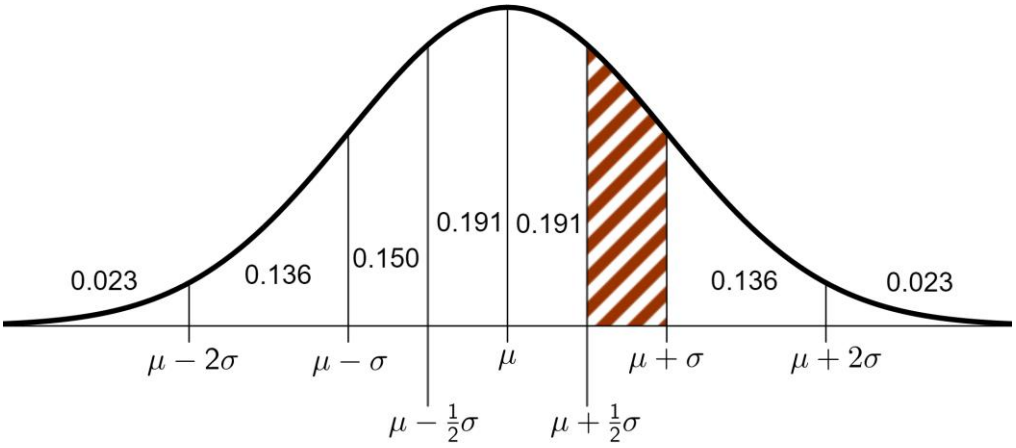
Take a new answer sheet for every question!

An exam consists of open questions and multiple choice questions.

The time that candidates need to answer the open questions is normally distributed with an average of 50 minutes and a standard deviation of 5 minutes.

There are 2500 candidates taking this exam.

4pt a Use the figure below to compute the expected number of candidates that have not finished answering the open questions after 60 minutes.



A normal probability distribution X

The area of the shaded region represents $P\left(\mu + \frac{1}{2}\sigma < X < \mu + \sigma\right) = 0.150$

There are six multiple choice questions. For each of these questions, the time that candidates need to answer the question is normally distributed with an average of 10 minutes and a standard deviation of 2 minutes.

3pt b Compute the probability that a candidate needs more than 60 minutes to answer all 6 multiple choice questions.

4pt c Compute the standard deviation of the total time that candidates need to answer all questions (open + multiple choice) of the exam.

Three of the multiple choice questions have 3 options for the answer, the other three have 4 options for the answer. Sandra answers all multiple choice questions randomly.

5pt d Compute the probability that Sandra answers precisely one of the multiple choice questions correctly.

Question 5 – Ebb and Flow

Take a new answer sheet for every question!

The water height in the Westerschelde (an estuary in the province of Zeeland) fluctuates under the influence of ebb and flow.

For a certain week , the water height can be approximated with the formula

$$H = 25 + 175\sin(0.5t)$$

In this formula, H is the water height in centimetres above average sea level and t is the time in hours, with $t = 0$ on Sunday midnight. Note that $0.5t$ is in radians.

2pt a Compute the minimal and the maximal water height.

4pt b Compute algebraically by how many percent the water height rises between Sunday, 1 AM ($t = 1$) and 2 AM ($t = 2$).

4pt c Compute algebraically the first two times after $t = 1$ at which the water height is the same as on $t = 1$.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\big(g(x)\big)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\big(g(x)\big) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^g\log a + {}^g\log b = {}^g\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a - {}^g\log b = {}^g\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a^p = p \cdot {}^g\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^g\log a = \frac{{}^p\log a}{{}^p\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ with $k = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$ $g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$ $g_r = \mu_T + 2.58\sigma_T$

