CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 21 April 2021
Time: 13.30 – 16.30
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c d	6 6 5	3 5 5	4 4 5 3	4 2 4 2	5 5 4	5 4
Total	17	13	16	12	14	9
Grade $= \frac{\text{total points scored}}{9} + 1$ You will pass the exam if your grade is at least 5.5 .						

Question 1 – Algebraic computations

Take a new answer sheet for every question!

When you are asked to perform a computation ***algebraically****, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$.
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = 2x^4 - 8x^3 + 9x^2 + 5x - 7$.
The line ℓ is given by the equation $y = 5x + 4$.

6pt a Compute algebraically the values of a for which the tangent line of the graph of f in the point $(a, f(a))$ is parallel to line ℓ.

The function g is given by $g(x) = 3\sqrt{4x^2 - 7}$

6pt b Compute algebraically the values of x for which the slope of the graph of g in the point $\left(x, g(x)\right)$ is equal to 8.

4pt c Compute algebraically the *x*-coordinate of the intersection of the graphs of $h(x) = 3 \cdot 2^x$ and $k(x) = \frac{1}{3} \cdot 4^x$.
Give an approximation of the answer rounded to 4 digits behind the decimal point.

Question 2 – Do not drink and drive!

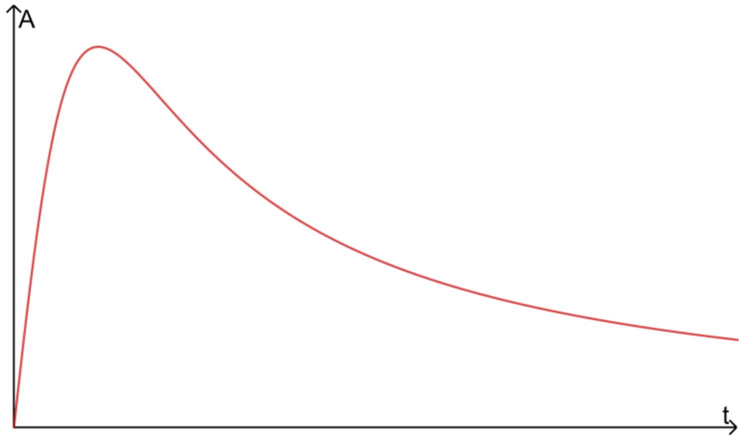
Take a new answer sheet for every question!

A test subject drinks a number of glasses of spirits in a very short time. The alcohol content is then measured in the air he exhales. This alcohol content increases rapidly immediately after drinking the spirits, but decreases again after some time. The relationship between the alcohol content and the time is approximated by the formula

$$A = \frac{4510\,t}{2t^2 + 5}$$

In this formula, A is the alcohol content in the exhaled air, measured in microgram per litre ($\mu g/l$), and t is the time in hours, with $t = 0$ at the time the test subject drinks the spirits.

In the figure below, the graph is shown that represents the relationship between A and t.



3pt a Compute algebraically by how many percent the alcohol content increases between $t = \frac{1}{2}$ and $t = 1$ according to the formula given above.
Give your answer rounded to a whole number.

In the Netherlands, a person is not allowed to participate in traffic if the alcohol content in the exhaled air is higher than 220 micrograms per litre.

5pt b Compute algebraically the first time after $t = 1$ at which the test subject is allowed to participate in traffic again according to the above formula.

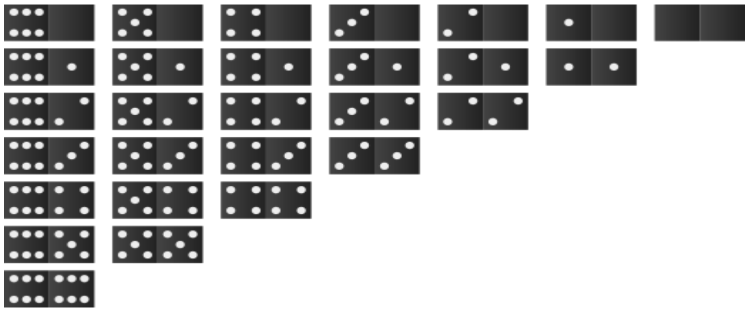
5pt c Compute algebraically the time at which the alcohol content in the exhaled air is maximal according to the above formula.
Give the answer rounded to whole minutes.

Question 3 – Dominoes

Take a new answer sheet for every question!

Dominoes is a family of tile-based games played with gaming pieces, commonly known as dominoes. Each domino is a rectangular tile, usually with a line dividing its face into two square *ends*. Each end is marked with a number of spots or is blank. The gaming pieces make up a domino set (source: Wikipedia).

The traditional European domino set consists of 28 tiles, each end containing 0 – 6 spots, see the figure below.



As you can see, there are 7 *double* dominoes, that are tiles with the same number of spots on each end (6-6, 5-5, 4-4, 3-3, 2-2, 1-1 and 0-0).

Two players, Yassine and Zaher, are playing with a traditional European domino set. At the start of the game, the dominoes are on a table with the spots facing the table. Then each player randomly takes 7 dominoes.
The player that will start the game is determined as follows:

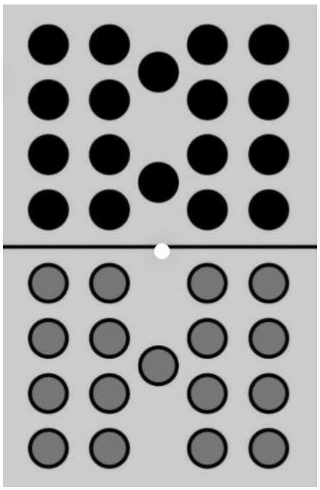
1. If both players have a double domino, then the player with the highest number of spots on the double domino will start (so 5-5 precedes 3-3).
2. If only one of the players has a double domino, the player having the double domino will start.
3. If neither player has a double domino, all dominoes are put back on the table and both players again randomly take 7 dominos.

The probability that neither player has a double domino is, rounded to 4 digits behind the decimal point, equal to 0.0029.

4pt a Show this with a computation.

4pt b Compute the probability that Yassine can start the first game immediately (that is without having to take dominoes for a second time).

Question 3 continued



In addition to the traditional European domino sets, there are also versions with 0 – 9 spots and with 0 – 18 spots.

Just like in the traditional European version, a set contains all combinations of 0 – 9 respectively 0 – 18 spots and of all possible double dominoes. As an example, the domino with 18 and 17 spots from the version with 0 – 18 spots is shown in the figure on the right.

5pt c Compute the number of dominoes in the version with 0 – 18 spots.

We now take a set dominos with 0 – 9 spots

3pt d Compute the number of dominoes in this set on which the total number of spots (the sum of the spots on both ends) is larger than 12.

Questions 4, 5 and 6 are on the next pages

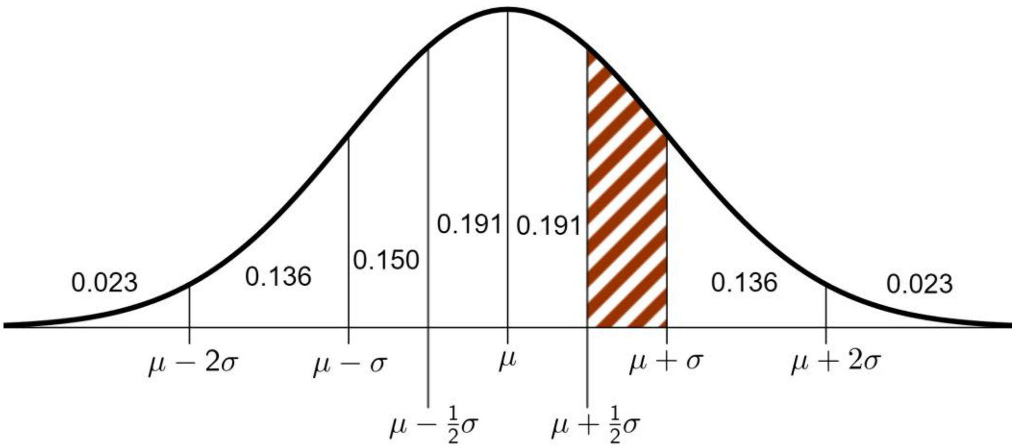
Question 4 – The weight of the domino tiles

Take a new answer sheet for every question!

The weight of the wooden domino tiles from the Onimod factory is normally distributed with an average of 5.4 grams and a standard deviation of 0.2 grams.

William randomly takes seven tiles from a domino set of 28 tiles.

4pt a Use the figure below to compute the probability that each of the seven tiles that William takes has a weight between 5.3 grams and 5.8 grams.



A normal probability distribution X
The area of the shaded region represents $P\left(\mu + \frac{1}{2}\sigma < X < \mu + \sigma\right) = 0.150$

During a quality check of the machine that produces the domino tiles, it is tested whether the average weight of the tiles is still 5.4 grams. For this, the weight of 36 tiles produced by this machine is measured. In this testing procedure it is assumed that the standard deviation of the weight of the tiles is still 0.2 g and a significance level of $\alpha = 0.05$ is taken.

2pt b State the null hypothesis and the alternative hypothesis for this test procedure.

The average weight of the 36 tiles is 5.46 g.
This results in a *p*-value of 0.03593.

4pt c Compute the parameters μ and σ of the test statistic that was used to compute this *p*-value.

2pt d What is the conclusion of this test procedure?
Explain your answer!

Question 5 – Air pressure

Take a new answer sheet for every question!

The air pressure decreases approximately exponentially with increasing altitude. The air pressure is measured in hectoPascal (hPa).

At a certain time an air pressure of 890 hPa is measured at an altitude of 1 km and an air pressure of 542 hPa is measured at an altitude of 5 km.

5pt a For this situation, compute algebraically the air pressure at a height of 0 km.

In addition to altitude, temperature also affects air pressure.
In the remainder of this question we assume that the air pressure is given by the formula

$$P = 1013 \cdot \mathrm{e}^{\frac{-0.034h}{t+273.15}}$$

In this formula, P is the air pressure in hPa, t is the air temperature in degrees Celsius (°C) and h is the height in in meters.
At a certain time, the air temperature at 1 km (= 1000 meter) altitude is 0°C.

5pt b Use the derivative $\frac{\mathrm{d}P}{\mathrm{d}h}$ to compute with how many hPa/meter the air pressure decreases at an altitude of 1 km at an air temperature of 0°C.

Suppose someone is in a hot air balloon with a barometer (= air pressure gauge) and a thermometer (= temperature gauge). The height can then be calculated using the given formula.

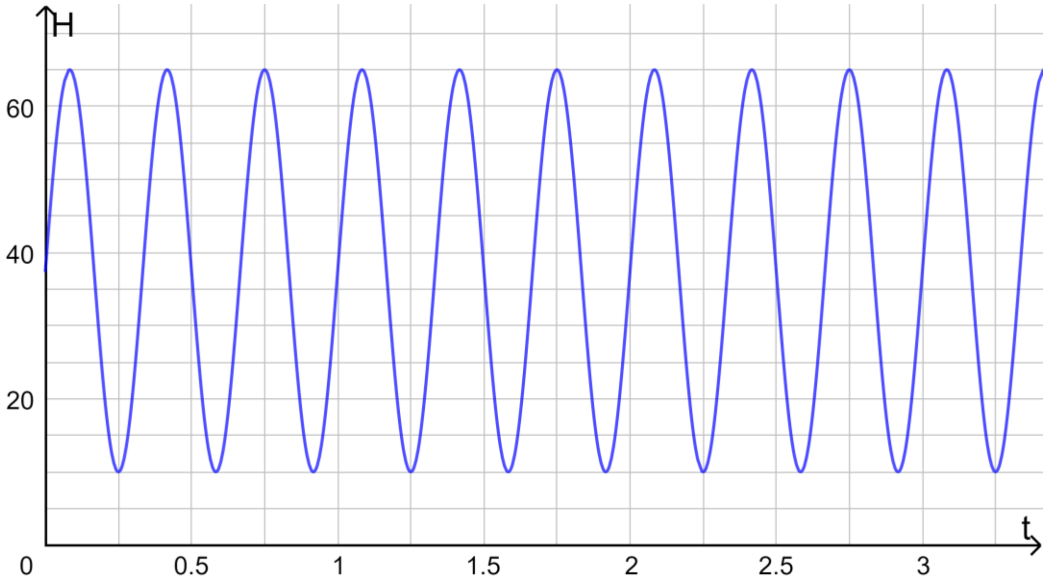
4pt c Rewrite the given formula into a formula in which the height h is expressed in the air pressure P and the air temperature t.

Question 6 – Electrical bicycles

Take a new answer sheet for every question!

In the Netherlands, the use of bicycles with an electric motor that provides pedal assistance (usually called an e-bike) has become very popular in recent years. The support provided by the motor depends on the muscle force exerted and the speed of the bicycle. This speed is measured using a sensor mounted on one of the spokes of the rear wheel, see the photo on the right.

On a nice day, Bert rides with a constant speed with the wind on a long straight road. The graph below shows the relationship between H, the height of the sensor above the road in cm and t, the time in seconds.



This graph fits a formula of the form $H = a + b\sin(ct)$.

5pt a Compute the values of a, b and c in this formula.

On the way back Bert has the wind against him, so he rides at a lower, but still constant speed. At this speed, $c = 16$.

The bicycle travels 235 cm per revolution of the rear wheel.

4pt b Compute algebraically the speed in km/hour at which Bert then cycles.
Give your answer rounded to one digit behind the decimal point.
(1 hour = 3600 seconds, 1 km = 100 000 cm)

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^{g}\log a + {}^{g}\log b = {}^{g}\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a - {}^{g}\log b = {}^{g}\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a^p = p \cdot {}^{g}\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^{g}\log a = \frac{{}^{p}\log a}{{}^{p}\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$$P(X=k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \quad \text{with} \quad k = 0, 1, 2, \ldots, n$$

Expected value: $E(X) = np$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$
			$g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$
			$g_r = \mu_T + 2.58\sigma_T$

