CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date: 17 December 2021
Time: 13.30 – 16.30
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a b c d	5 6 4	4 5 5	4 2 4 2	5 5 5	4 5 5 3	2 2 4
Total	15	14	12	15	17	8
Grade $= \frac{\text{total points scored}}{9} + 1$ You will pass the exam if your grade is at least 5.5 .						

Question 1 – Algebraic computations

Take a new answer sheet for every question!

*When you are asked to perform a computation **algebraically**, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$*.*
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = x^3 - 3x^2 + 4$.
The line ℓ is given by the equation $y = -\frac{9}{4}x + 4$.

5pt a Compute algebraically the coordinates of the common points of the graph of f and the line ℓ.

The function g is given by $g(x) = \sqrt{2x^2 + 1}$.

6pt b Compute algebraically the values of x for which $g'(x) = \frac{4}{3}$.

The relation between the variables *D* and *P* is given by the formula

$$D = 3 + 2\mathrm{e}^{P-1}$$

This formula can be rewritten into a formula of the form

$$P = a + \ln(D - b)$$

4pt c Compute algebraically the values of a and b in the rewritten formula.

Question 2 – Corona information by the government

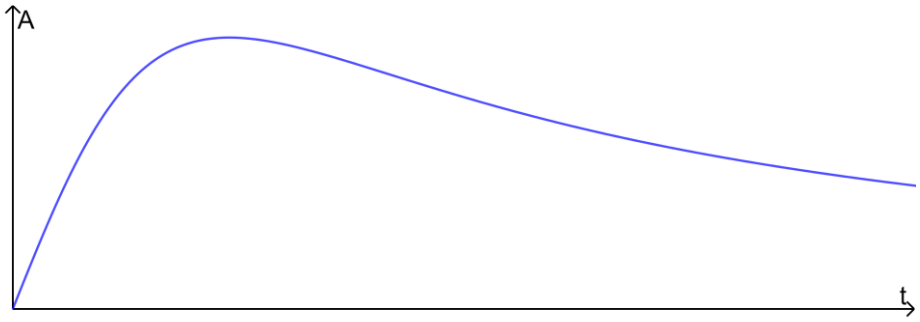
Take a new answer sheet for every question!

In country C, the government regularly gives press conferences about the corona measures. The details of these measures will be published on a special website immediately after the press conference. The number of visitors to this website is approximated by the formula

$$A = \frac{5000t}{16 + t^2}$$

In this formula, A is the number of visitors logged into the website in thousands and t is the time in minutes, with $t = 0$ when the website goes online.

In the figure below you can see the graph showing the relationship between A and t.



If more than 500,000 visitors are logged in, the website is overloaded. Visitors can still log in, but it takes a long time before the requested information is displayed.

4pt a Compute algebraically the number of minutes during which the website is overloaded according to the formula given above.

5pt b Compute algebraically the maximum number of visitors logged into the website according to the formula given above.

The number of visitors that are logged into the website can also be modelled with the formula

$$B = t \cdot \mathrm{e}^{-0.25t+6.05}$$

In this formula, B is the number of visitors logged into the website in thousands and t is again the time in minutes, with $t = 0$ when the website goes online.

5pt c Use the derivative $\frac{\mathrm{d}B}{\mathrm{d}t}$ to investigate whether, according to this second formula, the number of visitors logged into the website is still increasing at $t = 5$.

Question 3 – Little owls in the Achterhoek and in the Liemers

Take a new answer sheet for every question!

The little owl is a small brown-grey stocky owl about 21 to 27 cm long and is the smallest owl in the Netherlands after the pygmy owl. The little owl only breeds once a year from late March to June.

A breeding attempt is considered successful if at least one young is born.

A study in the Achterhoek, a region in the east of the province of Gelderland, shows that the probability of a successful breeding attempt in that region is equal to $\frac{3}{4}$.

One in 10 little owl pairs makes a second breeding attempt after a failed breeding attempt, but no pair makes a third attempt.

4pt a Compute the expected number of pairs with a successful breeding attempt in a population of 284 little owl pairs in the Achterhoek.

The Liemers is a region in Gelderland just southwest of the Achterhoek. Researchers want to test whether the probability of a successful breeding attempt in this region is also $\frac{3}{4}$. For this test procedure, a significance level of $\alpha = 0.05$ is used.

2pt b State the null hypothesis and the alternative hypothesis for this test procedure.

In the Liemers, 100 randomly selected breeding attempts are examined.

4pt c Compute the probability that exactly 70 of these 100 breeding attempts are successful if the probability of a successful attempt in this region is $\frac{3}{4}$.

2pt d Can you draw a conclusion for this test procedure based on your answer of item c? If so, state and motivate this conclusion. If not, explain why not.

Question 4 – Little owls throughout the Netherlands

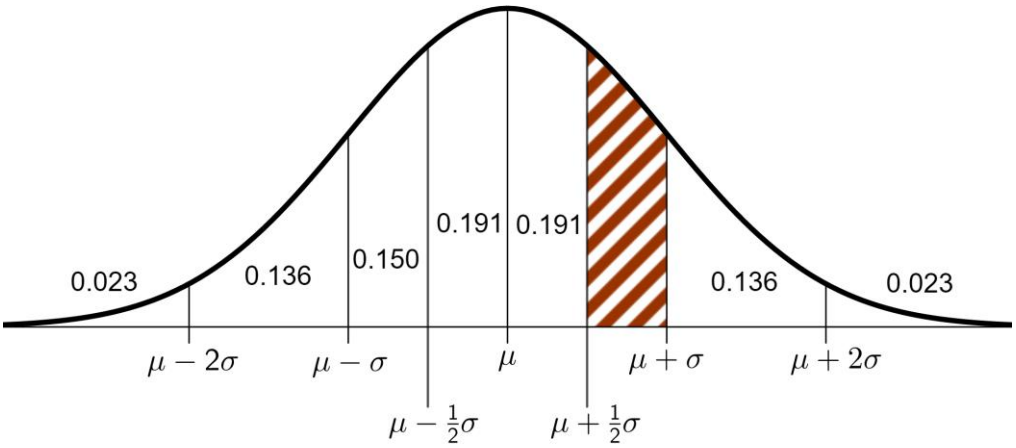
Take a new answer sheet for every question!

In 2018, it was established on the basis of extensive counts throughout the Netherlands that the percentage of successful broods per population of little owls was normally distributed. In 2.5% of the populations this percentage was less than 62.6. In 97.5% of the populations this percentage was less than 69.

5pt a Using the rules of thumb, calculate the probability that the percentage of successful broods in a population was less than 64.2.

The weights of both male and female little owls are normally distributed. Males weigh an average of 180 grams with a standard deviation of 6 grams, females weigh an average of 200 grams with a standard deviation of 8 grams.

5pt b Use the figure below to compute the probability that a breeding pair has a weight between 385 and 400 grams.



A normal probability distribution X
The area of the shaded region represents $P\left(\mu + \frac{1}{2}\sigma < X < \mu + \sigma\right) = 0.150$

After a young has hatched from the egg, it remains in the nest for more than a month until it fledges. In this period, a young has a 0.99 probability of surviving until the next day.
Two young hatched from the egg of a breeding pair on the same day.

5pt c Compute the probability that exactly one of these two young is still alive after 30 days.

Question 5 – Bicycle helmets

Take a new answer sheet for every question!

In country C, work is underway to improve road safety.
One of the measures taken is the mandatory wearing of helmets for cyclists. After the announcement of this helmet requirement, the percentage of cyclists wearing a helmet has increased significantly. On the day the measure is announced, 4% of cyclists wear a helmet, two weeks later 9% of cyclists wear a helmet.

4pt a Compute algebraically how long after the helmet requirement has been announced 50% of cyclists wear a helmet if this percentage increases **linearly**. Give your answer rounded to whole days.

5pt b Compute algebraically how long after the helmet requirement has been announced 50% of cyclists wear a helmet if this percentage increases **exponentially**. Again, give your answer rounded to whole days.

Over time, the percentage of cyclists wearing helmets will no longer increase linearly or exponentially. A formula with which this is well modelled is

$$p = \frac{180}{2 + 43 \cdot 2^{-t}}$$

In this formula, p is the percentage of cyclists who wear helmets and *t* is the time in weeks, with $t = 0$ at the time the measure is announced.

5pt c Compute algebraically the time when 50% of the cyclists wear a helmet according to this formula.

3pt d According to this formula, will all cyclists in country C wear helmets in the long run? If so, explain why. If not, what percentage of cyclists will not wear a helmet in the long run according to this formula?

Question 6 – The harbour of Herm

Take a new answer sheet for every question!

Herm is one of the smaller Channel Islands, mainly visited by day trippers who come by ferry from the nearby island of Guernsey. The water level in the harbour of Herm is of course dependent on the tide. For a certain day this water level is given by the formula

$$H = 4 + 3.5\,sin\left(\frac{1}{2}t\right)$$

In this formula, t is the time in hours with $t = 0$ at midnight and H is the water level in meters.

2pt a What is the minimum and maximum water level in the harbour of Herm on this day?

The period of H, rounded to whole minutes, is equal to 754.

2pt b Show this with a computation.

If the water level in the harbour is less than 1.5 meters, the ferries cannot dock in the harbour. They then divert to a jetty on the coast.
On the day mentioned above, the water level drops below 1.5 meters for the first time at 07:52 am (8 minutes to 8).

4pt c Compute the first time after 7:52 am when the ferries can dock again in the harbour because the water level rises above 1.5 meters again.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\big(g(x)\big)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\big(g(x)\big) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^g\log a + {}^g\log b = {}^g\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a - {}^g\log b = {}^g\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^g\log a^p = p \cdot {}^g\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^g\log a = \frac{{}^p\log a}{{}^p\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$ $\qquad$ $\sigma(S) = \sqrt{n} \cdot \sigma(X)$

$E(\overline{X}) = E(X)$ $\qquad$ $\sigma(\overline{X}) = \dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ with $k = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$ $\qquad$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$ $g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$ $g_r = \mu_T + 2.58\sigma_T$

