CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Program of the entrance exam Wiskunde B

Valid from December 2018

The entrance exam Wiskunde B is taken as a written examination with open questions. The exam time is 3 hours. Information about the exam dates and registration for these exams can be found on www.ccvx.nl.

The program of the entrance exam Wiskunde B of the CCVW is based on the program wiskunde B of the Dutch VWO for 2019 as published on www.examenblad.nl . Important difference: **graphing calculators** or any other ICT devices **may not be used** at the entrance exam Wiskunde B. The further determination of the VWO examination program on www.examenblad.nl is therefore not applicable.

In this document you will find

- The exam program
- The formula list that is printed on the exam
- Exam supplies
- Elaboration of the exam program in a list of concepts, characteristics and skills
- Overview of algebraic skills
- Recommended learning materials

In entrance exam all calculations must be performed algebraically, the use of a **graphing calculator** or a calculator with the possibility to calculate integrals is therefore **not allowed**.
The use of a standard calculator with exponential, logarithmic and trigonometric functions of a type similar to the Casio fx 82 series and the TI 30 series is permitted.

Exam program

This is a translation of the official program in Dutch. If there are any discrepancies between the text below and the Dutch version of this text, the Dutch version will prevail.

1 The candidate can analyse suitable problem situations in mathematical terms, solve them and translate the result back to the relevant context.

2 The candidate masters the mathematical skills appropriate to the examination program, including modelling and algebraizing, organizing and structuring, analytical thinking and problem solving, manipulating formulas, abstracting, and logical reasoning and proofs.

3 The candidate can interpret and edit formulas, draw a graph in a coordinate system in a relationship between two variables and determine whether a given formula can be rewritten as a function rule.

4 The candidate can draw and recognize graphs of the following standard functions: power functions with rational exponents, exponential functions, logarithmic functions, trigonometric functions and the absolute value function and can name and use the characteristic features of these different types of functions.

5 The candidate can draw up function rules, edit them, combine them, draw the corresponding graphs and make qualitative statements about the function and its graph on the basis of a function rule.

6 The candidate can conceptually use, draw up and use the inverse of a function.

7 The candidate can solve equations, inequalities and systems of two linear equations and interpret the solutions.

8 The candidate can determine the asymptotic behaviour of functions and can prove this behaviour with calculation of limits.

9 The candidate can interpret the first and second derivative of a function conceptually, can use these to examine the function and can use these in applications.

10 The candidate can determine the first and second derivative of functions using the rules for differentiation and can thereby use algebraic techniques.

11 The candidate can find the antiderivatives of the standard functions mentioned under 4 and simple combinations thereof (direct integration) and can draw up a definite integral and calculate this exactly in suitable applications.

12 The candidate can draw up and edit formulas for periodic phenomena, draw the corresponding graphs, solve equations and use the periodicity with insight, using the formula list where necessary.

13 The candidate can investigate and prove geometrical properties of objects, using geometric and algebraic techniques when necessary.

14 The candidate can examine the properties and mutual location of points, lines, circles and other suitable figures by means of algebraic representations, can form algebraic representations of figures in a given or self-chosen coordinate system and can use algebraic representations to solve geometrical problems.

15 The candidate can derive properties from figures in the plane with the help of vectors and in-products and perform calculations.

16 The candidate can apply the indicated techniques in suitable scientific and technical situations.

Formula list

This list will be printed on the last page of the entrance exam Wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t+u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t-u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t+u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t-u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2\sin(t)\cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$$

Aids required during the entrance exams

To take the exam you must take:

- Identity document. *Your identity document will be inspected during the entrance exam. You should always be able to identify yourself with a valid identity document (passport, Dutch driving licence, Dutch identity card, EU/EEA document, residence document model 2001 types I to IV).*
- Writing utensils*. A pen (not a red one) and a pencil. A pencil may only be used for drawing graphs.*
- A set square and protractor.
- calculator with exponential, logarithmic and goniometric functions. *A **graphic calculator** and\or a calculator with the ability to calculate integrals is **not permitted**; a list of formulae will be provided with the exam. Other aids, such as a formula sheet, BINAS and table book are NOT permitted.*
- Watch. *It might be a good idea to bring a watch (no smartwatch) to be able to divide your time over the different assignments. You are not allowed to use your mobile phone as a watch.*
- It may also be wise to bring spare batteries for your calculator

Make sure you take the correct calculator with you. If you only have a graphing calculator with you, you will have to take the exam without a calculator.

Elaboration of the exam program

Below, the exam program is further elaborated in a list of concepts, properties and skills. This list is intended as a support in preparing for the entrance exam, but not as a replacement for the exam program. Although this list has been compiled with the greatest possible care, it may therefore occur that an exam question is not dealt with in this list.

Concept / property / skill	Remark / explanation
Standard functions	
Power functions: $f(x) = x^n$	n is a rational number (a fraction)
Exponential functions: $f(x) = a^x$	$a > 0$
Logarithmic functions: $f(x) = {}^{g}\log(x)$	$g > 0$; $g \neq 1$; $x > 0$
The number e and the natural logarithm	$\mathrm{e} \approx 2.718282$; ${}^{\mathrm{e}}\log(x) = \ln(x)$
Trigonometric functions: $f(x) = \sin(x)$; $g(x) = \cos(x)$	$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$
Absolute value function: $f(x) = \|x\|$	$\|x\| = x$ for $x \geq 0$ $\|x\| = -x$ for $x < 0$
Combining functions	
Functions can be added and can be multiplied by a constant	$f(x) = 4x^4 + 3x^3 + 2x^2 + x + 1$ is a combination of 5 power functions
Functions can be multiplied and divided	For example $h(x) = x^2 \cdot \mathrm{e}^x$
Composition of functions $h(x) = f(g(x))$	$f(x) = \mathrm{e}^x$ and $g(x) = x^2$ yields $h(x) = \mathrm{e}^{x^2}$
Inverse functions $y = f(x) \Leftrightarrow x = f^{inv}(y)$	$f(x) = x^3$ yields $f^{inv}(y) = \sqrt[3]{y}$ $f(x) = g^x$ yields $f^{inv}(y) = {}^{g}\log(y)$

Domain and Range	
Domain: al permitted values of *x*	$f(x) = x^{-1} = \frac{1}{x}$ does not exist if $x = 0$
Range: all possible values of *y*	$f(x) = x^2$ yields $y = x^2 \geq 0$
Domain and range are often given in the interval notation	You must be able to read this, but you do not have to use it yourself

Powers	
a^n is a power with base a and exponent n	n is a rational number (a fraction)
Special exponents	
$a^2 = a \cdot a;\quad a^3 = a \cdot a \cdot a$ etc.	
$a^1 = a$	
$a^0 = 1$	$a \neq 0$
$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$	$a \neq 0$ $a^{-1} = \frac{1}{a^1} = \frac{1}{a}$
$a^{\frac{1}{n}} = \sqrt[n]{a}$	$n = 2, 3, 4, 5, \ldots$ $a \geq 0$ for even values of n $a^{\frac{1}{2}} = \sqrt[2]{a} = \sqrt{a}$
Calculation rules for powers	
$a^m \cdot a^n = a^{m+n}$	
$\frac{a^m}{a^n} = a^{m-n}$	
$(a^m)^n = a^{m \cdot n}$	$a^{\frac{m}{n}} = a^{\frac{1}{n} \cdot m} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$
$(a \cdot b)^n = a^n \cdot b^n$	$(a+b)^2 \neq a^2 + b^2$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

Power functions	
Standard function: $f(x) = x^n$	The input variable x is the base
Constant function: $f(x) = c$	$f(x) = c \cdot x^0$ The graph is a horizontal line
Lineair functions: $f(x) = ax + b$	$f(x) = a \cdot x^1 + b \cdot x^0$ The graph is the straight line with equation $y = ax + b$
Quadratic functions: $f(x) = ax^2 + bx + c$	The graph is a parabola with axis of symmetry $x = -\frac{b}{2a}$ The graph is opening upward if $a > 0$ and downward if $a < 0$ The number if intersections with the *x*-axis is determined by $D = b^2 - 4ac$
Root and radical functions:	
For $x \geq 0$, $f(x) = x^{\frac{1}{2}} = \sqrt{x}$ is the inverse function of $f(x) = x^2$	Domain: $x \geq 0$; Range: $y \geq 0$ The graph is a half (horizontally orientated) parabola
$f(x) = x^{\frac{1}{3}} = \sqrt[3]{x}$ is the inverse function of $f(x) = x^3$	Domain: $\mathbb{R}$; Range: $\mathbb{R}$

Exponential functions	
$f(x) = a^x$ with $a > 0$	The input variable x is the exponent If $a > 1$, $f(x)$ is a convex increasing function with a horizontal asymptote for $x \to -\infty$ If $0 < a < 1$, $f(x)$ is a convex decreasing function with a horizontal asymptote for $x \to \infty$
Exponential growth formula with initial value *b* and growth factor *g*	$N(t) = b \cdot g^t$
Doubling time, half-life	

Logarithmic functions	
$l(x) = {}^{g}\log(x)$ is the inverse function of $f(x) = g^x$	$y = {}^{g}\log(x) \Leftrightarrow g^y = x$ $\mathrm{e}^x = y \Leftrightarrow x = {}^{\mathrm{e}}\log(y) = \ln(y)$
${}^{g}\log(1) = 0$; ${}^{g}\log(g) = 1$	$\ln(1) = 0$; $\ln(\mathrm{e}) = 1$
$\log(x) = {}^{10}\log(x)$	$\log(1) = 0$; $\log(10) = 1$
Domain: $x > 0$	The graph has the line $x = 0$ as its vertical asymptote For $g > 1$ the graph is concave and increasing For $0 < g < 1$ the graph is convex and decreasing
Calculation rules for logarithms	These follow from the calculation rules for powers
${}^{g}\log(a) + {}^{g}\log(b) = {}^{g}\log(a \cdot b)$	
${}^{g}\log(a) - {}^{g}\log(b) = {}^{g}\log\left(\frac{a}{b}\right)$	
${}^{g}\log(a^n) = n \cdot {}^{g}\log(a)$	This holds for *all* numbers n
${}^{g}\log(a) = \frac{{}^{p}\log(a)}{{}^{p}\log(g)}$	Often used with $p = 10$ or $p = \mathrm{e}$ to compute logarithms with the log or ln function of a calculator

Trigonometry	
Sine, cosine and tangent in a right-angled triangle	
Angles in a unit circle	
Angles in degrees and in radians	$180° = \pi\, rad$; $1° = \frac{\pi}{180} rad$
Exact values of the sine and the cosine of standard angles	30-60-90 triangle 45-45-90 triangle
Pythagorean identity	$\cos^2(\alpha) + \sin^2(\alpha) = 1$
Sum formulas	*In the formula list*

Trigonometric functions $f(x) = \sin(x);\ \ g(x) = \cos(x)$	Computations with trigonometric functions are usually in radians
The period of these standard functions	2π
Minima	$\sin(x) = -1$ for $x = -\frac{1}{2}\pi + k \cdot 2\pi$ $\cos(x) = -1$ for $x = \pi + k \cdot 2\pi$
Maxima	$\sin(x) = 1$ for $x = \frac{1}{2}\pi + k \cdot 2\pi$ $\cos(x) = 1$ for $x = 0 + k \cdot 2\pi$
Zeros	$\sin(x) = 0$ for $x = 0 + k \cdot \pi$ $\cos(x) = 0$ for $x = \frac{1}{2}\pi + k \cdot \pi$
Shape of the graph	Sinusoid
Symmetry relations	$\sin(x) = \sin(\pi - x)$ $\cos(x) = \cos(-x)$ $-\sin(x) = \sin(-x) = \sin(\pi + x)$ $-\cos(x) = \cos(\pi - x) = \cos(\pi + x)$
Horizontal translations	$\sin(x) = \cos\left(x - \frac{1}{2}\pi\right)$ $\cos(x) = \sin\left(x + \frac{1}{2}\pi\right)$
General form of a sinusoid $f(x) = A + B \cdot \sin\big(C \cdot (x - D)\big)$	$A =$ equilibrium $\lvert B \rvert =$ amplitude $C = \dfrac{2\pi}{\text{period}}$ If $B > 0$, the graph passes the equilibrium in point (D, A) (starting point) and f is increasing in this point
Simple harmonic motion $u = a \cdot \sin\left(\frac{2\pi}{T} \cdot t\right)$	$a =$ amplitude $T =$ time period $f = \frac{1}{T} =$ frequency

Differentiation Derivatives of standard functions Various notations for the derivative	$f(x) = c$ yields $f'(x) = 0$ $f(x) = x^a$ yields $f'(x) = ax^{a-1}$ $f(x) = a^x$ yields $f'(x) = a^x \cdot \ln(a)$ $f(x) = \mathrm{e}^x$ yields $f'(x) = \mathrm{e}^x$ $f(x) = \ln(x)$ yields $f'(x) = \frac{1}{x}$ $f(x) = \sin(x)$ yields $f'(x) = \cos(x)$ $f(x) = \cos(x)$ yields $f'(x) = -\sin(x)$
Rules for differentiation of combinations of functions Constant factor rule Sum rule Product rule Quotient rule Chain rule	$f(x) = {}^g\log(x) = \frac{\ln(x)}{\ln(g)} = \frac{1}{\ln(g)} \cdot \ln(x)$ yields $f'(x) = \frac{1}{\ln(g)} \cdot \frac{1}{x}$ Use the quotient rule to differentiate $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

Antidifferentiation and integration $F(x)$ is antiderivative function of $f(x)$ if $F'(x) = f(x)$ If $F(x)$ ean antiderivative of $f(x)$, then $G(x) = F(x) + C$ is an antiderivative of $f(x)$ too. Definite integral: $$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$ Definite integral and area	 This is because the constant C vanishes if we differentiate $G(x)$ In this formula, $F(x)$ is an antiderivative of $f(x)$ If $f(x) \geq 0$ on the interval $a \leq x \leq b$, then the area of the region bounded by the line $x = a$, the *x*-axis, the line $x = b$ and the graph of f is equal to $\int_a^b f(x)\,\mathrm{d}x$

Standard antiderivatives	
$f(x) = c$ yields $F(x) = cx + C$	
$f(x) = x^a$ yields $F(x) = \frac{1}{a+1}x^{a+1} + C$	$a \neq -1$
$f(x) = \mathrm{e}^x$ yields $F(x) = \mathrm{e}^x + C$	
$f(x) = \frac{1}{x}$ yields $F(x) = \ln(\lvert x \rvert) + c$	For $x < 0$ this reads $F(x) = \ln(-x) + C$
$f(x) = \sin(x)$ yields $F(x) = -\cos(x) + C$	
$f(x) = \cos(x)$ yields $F(x) = \sin(x) + C$	
Calculation rules for antidifferentiation of combinations of functions	
Constant factor rule	$g(x) = c \cdot f(x)$ yields $G(x) = c \cdot F(x)$
Sum rule	$s(x) = f(x) + g(x)$ yields $S(x) = F(x) + G(x)$
Simple chain rule	$g(x) = f(ax + b)$ yields $G(x) = \frac{1}{a} \cdot F(ax + b)$
Calculation rule for definite integrals	$\int_a^b f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x$ $(a < c < b)$
Applications of definite integrals	
Area of the region with upper boundary $f(x)$ and lower boundary $g(x)$	$\int_a^b f(x) - g(x)\,\mathrm{d}x$
Volume of a solid of revolution with the *x*-axis as the axis of revolution	$\pi \int_a^b y^2\,\mathrm{d}x$ with $y = f(x)$
Volume of a solid of revolution with the y-axis as the axis of revolution	$\pi \int_c^d x^2\,\mathrm{d}y$ with $x = f^{inv}(y)$
Definite integral with variable boundaries	$g(p) = \int_1^p f(x)\,\mathrm{d}x = F(p) - F(1)$

Geometry in a Cartesian coordinate system *In geometry with coordinates, we assume that the units on the axes are equal. This also holds when we examine graphs of functions geometrically.*	
Straight lines	
General form of the equation of a straight line: $mx + ny = c$	
Commonly used form: $y = ax + b$	Not for vertical lines
Slope: $\frac{\Delta y}{\Delta x} = \frac{y_B - y_A}{x_B - x_A}$	The slope is equal to a in the formula $y = ax + b$
The equation of the line through $(p, 0)$ and $(0, q)$ can also be written as $\frac{x}{p} + \frac{y}{q} = 1$	Not possible when $p = 0$ or $q = 0$
Straight lines and angles	
The direction angle of a straight line is the angle α between the line and the *x*-axis with $-90° < \alpha < 90°$	$\tan(\alpha) =$ slope
The angle β between two straight lines is the smallest of the angles between these lines in their intersection, so $0° \leq \beta \leq 90°$	This angle is determined by drawing both lines in a Cartesian plane and indicating the direction angles at the intersection.
Two lines are perpendicular if the product of their slopes equals -1	*The reverse is also true:* If two lines are perpendicular, then the product of their slopes equals -1

Perpendicular line and distance Perpendicular line	The slope of the line perpendicular to the line with equation $y = ax + b$ is $-\frac{1}{a}$
Distance between the points $A(a_1, a_2)$ and $B(b_1, b_2)$	$\|AB\| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2}$ also noted by $d(A, B)$
Projection of a point on a line Distance between a point and a line Distance between two parallel lines	Distances are measured along perpendicular lines. There is a formula for the distance between a point and a straight line which you may use. However, you do not need to memorize this formula.
Line segment bisector	The line segment bisector of the line segment *AB* consists of all points *P* for which $\|PA\| = \|PB\|$
Angle bisector	The pair of angle bisectors of the lines *l* and *m* consists of all points *P* for which $d(P, l) = d(P, m)$

Geometry in triangles	
Pythagorean theorem	
Area of a triangle	$\frac{1}{2} \times \text{base} \times \text{height}$
Sum of the angles	Is always equal to $180°$
Law of sines and law of cosines	
Isosceles triangles	The angles between the equal sides and the third side are also equal
Equilateral triangles	All angles are $60°$
Similar triangles	Two triangles are similar if the angles of the first triangle are equal to the angles of the second triangle. In that case, the ratios of the lengths of the sides are also equal
Median Centroid (centre of mass)	The centroid divides each median into two parts; the length of the longer part is twice the length of the shorter

Vectors	
A vector indicates a translation over a certain distance in a certain direction	So a vector $\vec{v}$ has a length, indicated by $\lvert\vec{v}\rvert$, and a direction
The vector that indicates the translation from point *A* to point *B* is noted by $\overrightarrow{AB}$	
Null vector $\vec{0}$	$\lvert\vec{0}\rvert = 0$; $\vec{0}$ is the only vector that has no direction
Vectors can be added	Head-tail method
Vectors can be multiplied by a constant	$\vec{b} = c \cdot \vec{a}$ with $c > 0$ has the same direction as $\vec{a}$; $\lvert\vec{b}\rvert = c \cdot \lvert\vec{a}\rvert$ $\vec{b} = c \cdot \vec{a}$ with $c < 0$ has the opposite direction of $\vec{a}$; $\lvert\vec{b}\rvert = -c \cdot \lvert\vec{a}\rvert$
Vectors with coordinates	
$\begin{pmatrix}4\\3\end{pmatrix}$ is a vector with coordinates 4 and 3	$\overrightarrow{OA} = \begin{pmatrix}4\\3\end{pmatrix}$ indicates the translation from $O(0,0)$ to $A(4,3)$
$\overrightarrow{OA}$ is the position vector of point *A*	
Computations with coordinates	
Addition	$\begin{pmatrix}4\\3\end{pmatrix} + \begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}4 + -1\\3 + 1\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}$
Multiplication by a constant	$-5 \cdot \begin{pmatrix}-1\\0{,}2\end{pmatrix} = \begin{pmatrix}-5 \cdot -1\\-5 \cdot 0{,}2\end{pmatrix} = \begin{pmatrix}5\\-1\end{pmatrix}$
The length of a vector	$\left\lvert\begin{pmatrix}4\\3\end{pmatrix}\right\rvert = \sqrt{4^2 + 3^2} = 5$
The coordinates of the null vector	$\vec{0} = \begin{pmatrix}0\\0\end{pmatrix}$
Decomposition along the axes	$\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix} + \begin{pmatrix}0\\3\end{pmatrix}$

Inner product $\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert \cdot \lvert\vec{b}\rvert \cdot \cos(\alpha)$	$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1 b_1 + a_2 b_2$ $\cos(\alpha) = \dfrac{a_1 b_1 + a_2 b_2}{\lvert\vec{a}\rvert \cdot \lvert\vec{b}\rvert}$
If two vectors are perpendicular, there inner product is 0	*The reverse is also true:* If the inner product of two vectors is 0, then they are perpendicular (except if one of the vectors is the null vector)
Vector representation of a line $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \lambda \begin{pmatrix} p \\ q \end{pmatrix}$ Position vector $\begin{pmatrix} a \\ b \end{pmatrix}$ Direction vector $\begin{pmatrix} p \\ q \end{pmatrix}$	A.k.a. parametric representation The number λ is called a parameter
A normal vector of a line is a vector perpendicular to the direction vector of the line	The line *l* with direction vector $\begin{pmatrix} p \\ q \end{pmatrix}$ has normal vector $\vec{n_l} = \begin{pmatrix} q \\ -p \end{pmatrix}$ The line *m* with equation $px + qy = c$ has normal vector $\overrightarrow{n_m} = \begin{pmatrix} p \\ q \end{pmatrix}$

Geometry in circles	
Equation of the circle with centre $M(a, b)$ and radius *r*: $(x-a)^2+(y-b)^2=r^2$	Conversion of the first to the second form by expanding brackets
The equation of a circle can also be written in the form $x^2+y^2+px+qy+s=0$	Conversion of the second to the first form by completing the square
Computing the intersection(s) with a straight line	Substitution will yield a quadratic equation with 0, 1 or 2 solutions
Tangent line, point of tangency	
A tangent line to a circle is perpendicular to the radius to the point of tangency	Slope tangent line × slope radius = -1
Computing the intersections of circles	Elimination of x^2+y^2 yields the equation of the line through the intersections
Touching circles	The common tangent line is perpendicular to the line through the centres
Tangent lines through a point *P* outside the circle	The points of tangency both have the same distance to *P*
Computing the distance between two circles	Using the line through the centres
Computing the distance between a line and a circle	Using the perpendicular line from the centre of the circle to the line
Triangles and circles	
The circumscribed circle of a triangle is the circle that passes through the three vertices	The centre is the intersection of the line segment bisectors of the sides
The inscribed circle of a triangle has all three sides as tangent lines	The centre is the intersection of the bisectors of the angles
Thales's theorem	If one of the sides of a triangle is a diameter of the circumscribed circle, the opposite angle is a right angle
Converse of Thales's theorem	In a right angled triangle, the midpoint of the hypotenuse is the centre of the circumscribed circle

Graphs Shape of the graphs of standard functions	
Drawing and sketching	When drawing a graph, the value of the numbers on the axes must be indicated, the coordinates of several points must be computed, asymptotes must be indicated if applicable and the domain and the range must be clearly visible. In a sketch of a graph, the shape of the graph, the minima and maxima and the asymptotes must be indicated.
Transformations Horizontal translation over c units Vertical translation over c units Horizontal multiplication Vertical multiplication Reflexion in the *y*-axis Reflexion in the *x*-axis	 $g(x) = f(x - c)$ $g(x) = f(x) + c$ $g(x) = f\left(\frac{1}{c} \cdot x\right)$ $g(x) = c \cdot f(x)$ $g(x) = f(-x)$ $g(x) = -f(x)$
Line symmetry in the *y*-axis	$f(-x) = f(x)$
Point symmetry in the origin	$f(-x) = -f(x)$
Vertical distance between two graphs	The distance between the intersections of the graphs of two functions with the vertical line $x = p$ is given by $\lvert f(p) - g(p) \rvert$

Asymptotes and limits	
Computing limits	Simple examples: $\lim_{x\to\infty}\frac{1}{x}=0$; $\lim_{x\to 2}\frac{2-x}{4-x^2}=\lim_{x\to 2}\frac{1}{2+x}=\frac{1}{4}$
Horizontal asymptote	When $\lim_{x\to\infty} f(x)=c$ or $\lim_{x\to-\infty} f(x)=c$
Vertical asymptote	When $\lim_{x\uparrow a} f(x)=\infty$ or $\lim_{x\uparrow a} f(x)=-\infty$ or $\lim_{x\downarrow a} f(x)=\infty$ or $\lim_{x\downarrow a} f(x)=-\infty$
Vertical asymptote for a quotient function	$q(x)=\frac{f(x)}{g(x)}$ has a vertical asymptote when $g(x)=0 \wedge f(x)\neq 0$
Oblique asymptote	When $\lim_{x\to\infty}(f(x)-(ax+b))=0$ or $\lim_{x\to-\infty}(f(x)-(ax+b))=0$
Perforation (removable discontinuity)	When $\lim_{x\uparrow a} f(x)=\lim_{x\downarrow a} f(x)$, but $f(a)$ does not exist
Jump discontinuity	When $\lim_{x\uparrow a} f(x)\neq\lim_{x\downarrow a} f(x)$

Applications of the derivative	
$f'(a)$ is the slope of the (tangent line to the) graph f in point $(a, f(a))$	$f'(a) =$ slope of the tangent line $= \tan(direction\ angle)$ in point $(a, f(a))$
The angle between two curves in a point *P*	This is the angle between the tangent lines to the curves is point *P*
When $f'(x) > 0, t$he graph is increasing; when $f'(x) < 0$, the graph is decreasing	
In a minimum and in a maximum we have $f'(x) = 0$ Minima and maxima together are called extremes. The points where the graph has a minimum or a maximum are called the vertices of the graph.	The points where f could have an extreme are computed by solving $f'(x) = 0$. After solving this, you have to find the minima and maxima with a sketch of the graph singular: vertex
Touching graphs	$f(x) = g(x)$ and $f'(x) = g'(x)$
Perpendicular graphs	$f(x) = g(x)$ and $f'(x) \cdot g'(x) = -1$
In a point where the derivative function has an extreme, the graph has a point of inflexion. In such a point of inflexion we have $f''(x) = 0$ When $f'(x) > 0$ and $f''(x) > 0$, the graph is increasing and convex (bending upward) When $f'(x) > 0$ and $f''(x) < 0$, the graph is increasing and concave (bending downward) When $f'(x) < 0$ and $f''(x) > 0$, the graph is decreasing and convex When $f'(x) < 0$ and $f''(x) < 0$, the graph is decreasing and concave	The points where f could have a point of inflexion are computed by solving $f''(x) = 0$. *After solving this, you should check whether the derivative indeed has an extreme in such a point, but usually this will be clear from the wording of the question*
Number of intersections of a graph with a horizontal line	Sketch the graph with special attention tot the vertices

Parametric equations	
Parametric equations determine the position of a point $P\big(x(t), y(t)\big)$ using a parameter *t*. When *t* represents time, the parametric equations represent a point moving along a curve, called the path of the point	Example: $\begin{cases} x(t) = t^2 \\ y(t) = 2t \end{cases}$ or as vector: $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$
Vertices of the path	Horizontal tangent line when $x'(t) \neq 0$ and $y'(t) = 0$ Vertical tangent line when $x'(t) = 0$ and $y'(t) \neq 0$
With both $x(t)$ and $y(t)$ linear functions, the path is a straight line	$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} at + b \\ ct + d \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} + t\begin{pmatrix} a \\ c \end{pmatrix}$
The path of a point *P* with parametric equations $\begin{cases} x(t) = a + r\cos(t) \\ y(t) = b + r\sin(t) \end{cases}$ Is a circle with centre $M(a, b)$ and radius r	So the equation of this circle is $(x - a)^2 + (y - b)^2 = r^2$
With both x and y periodical functions, the period of the movement of *P* is the least common multiple of the periods of x and y	
Velocity vector	$\vec{v} = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}$
Velocity in the *x*-direction	$\overrightarrow{v_x} = \begin{pmatrix} x'(t) \\ 0 \end{pmatrix}$
Velocity in the *y*-direction	$\overrightarrow{v_y} = \begin{pmatrix} 0 \\ y'(t) \end{pmatrix}$
Velocity of the moving point	$v = \lvert\vec{v}\rvert = \sqrt{\big(x'(t)\big)^2 + \big(y'(t)\big)^2}$
Slope of the tangent to the path in point *P*	$\frac{y'(t)}{x'(t)}$
Acceleration vector	$\vec{a} = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix}$
Acceleration of the moving point	$\frac{\mathrm{d}v}{\mathrm{d}t}$ with v the velocity of the movement

Algebraic skills

Below an overview of algebraic skills that the candidates for the examination mathematics A of the CCVW must master. Though this list has been compiled with the utmost care too, it may occur that a skill that is included the exam program, is not included in this list

Skill	Remark / explanation
Solving standard equations	
First degree	$ax + b = px + q$
Quadratic	$ax^2 + bx + c = 0$
Power equations with positive even exponents	$x^n = a \Leftrightarrow x = \pm\sqrt[n]{a} \quad (a \geq 0)$
Other power equations	$x^n = a \Leftrightarrow x = a^{\frac{1}{n}}$
Exponential and logarithmic equations that can be solved exactly	${}^{g}\log(g^n) = a \Leftrightarrow g^n = g^a \Leftrightarrow n = a$
Solving exponential equations using logarithms	$g^x = a \Leftrightarrow x = {}^{g}\log(a)$
Solving trigonometric equations exactly	$\sin(x) = \sin(A);\ \cos(x) = \cos(A);$ $\sin(x) = c$ en $\cos(x) = c$ met $c = 0, \pm\frac{1}{2}, \pm\frac{1}{2}\sqrt{2}, \pm\frac{1}{2}\sqrt{3}, \pm 1$
Solving trigonometric equations using inverse functions	$\sin(x) = c \Leftrightarrow \sin(x) = \sin(\sin^{-1}(c))$ $\cos(x) = c \Leftrightarrow \cos(x) = \cos(\cos^{-1}(c))$
Equations with absolute values	$\lvert x \rvert = a \Leftrightarrow x = \pm a \quad (a \geq 0)$
Solving inequalities	When solving inequalities graphically, the intersections have to be computed algebraically
Solving systems of equations	By elimination and / or substitution
Finding the equation of a straight line	Determine slope or normal vector and enter these together with the coordinates of a point in one of the standard formulas

Rewriting equations and formulas	*E.g. to rewrite an equation into a standard equation*
Splitting an equation	$A \cdot B = 0 \Leftrightarrow A = 0 \vee B = 0$ $A^2 = B^2 \Leftrightarrow A = \pm B$ $A \cdot B = A \cdot C \Leftrightarrow A = 0 \vee B = C$
Factorising using a common factor	$A \cdot B + A \cdot C = A(B + C)$
Expanding parentheses	$(A + B)(C + D) = AC + AD + BC + BD$
Operations with fractions	Addition (also of fractions with unlike denominators) Multiplication and division Crosswise multiplication: $\frac{A}{B} = \frac{C}{D} \Leftrightarrow AD = BC \wedge BD \neq 0$
Operations with roots	$\sqrt{A} = B \Leftrightarrow A = B^2 \wedge A \geq 0$
Solving equations by substitution	$x^4 + 4x^2 - 5 = 0$ with $y = x^2$ $\mathrm{e}^{2x} + 4\mathrm{e}^x - 5$ with $y = \mathrm{e}^x$ both yield $y^2 + 4y - 5 = 0$
Solving for a variable	Solve x from $y = f(x)$
Applying calculation rules and properties	For powers and logarithms and in trigonometric equations and functions
Substitution of formulas into other formulas	With correct use of brackets
Computations with parameters in function rules	
General	In (anti)differentiation, the parameter is a constant, in (systems of) equations, the parameter often is an extra unknown
The number of intersections with graphs of quadratic functions	$D = 0$ yields an equation for the parameter
Curve through the vertices of a graph	Eliminate the parameter from the system of equations $y = f_p(x)$ and $f_p'(x) = 0$
Touching graphs	$f_p(x) = g(x)$ and $f_p'(x) = g'(x)$

Recommended learning materials

Unfortunately, as far as we know there are no textbooks in English covering the full program of the Dutch VWO exam wiskunde B. However, there is a textbook that covers most of the calculus topics (functions, (anti)differentiation and integrals):

Mathematics that works volume 1 (De Gee, ISBN 978-90-5041-167-7)

For other topics, there is a wide range of materials available online, for instance the Kahn academy. You can use Google to find more information on a certain topic.

And last but not least, check the example exams on www.ccvx.nl